The Secure Boston Mechanism: Theory and Experiments*

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Abstract

This paper introduces a new matching mechanism that is a hybrid of the two most common mechanisms in school choice, the Boston Mechanism (BM) and the Deferred Acceptance algorithm (DA). BM is the most commonly used mechanism in the field, but it is neither strategyproof nor fair. DA is the mechanism that is typically favored by economists, but it is not Pareto efficient. The new mechanism, the Secure Boston Mechanism (sBM), is an intuitive modification of BM that secures any school a student was initially guaranteed but otherwise prioritizes a student at a school based upon how she ranks it. Relative to BM, theoretical results suggest that sBM is an improvement in terms of strategyproofness and fairness. We present experimental evidence using a novel experimental design that confirms that sBM significantly increases truth-telling and fairness. Relative to DA, theoretical results suggest that sBM *can* be a Pareto improvement in equilibrium but the efficiency comparison of sBM and DA is theoretically ambiguous. We present simulation evidence that suggests that sBM often *does* Pareto dominate DA when DA is inefficient, while sBM and DA very often overlap when DA is efficient. Overall, our results strongly support the use of sBM over BM and suggest that sBM should be considered as a viable alternative to DA.

Keywords: School choice, student assignment, preference manipulations, lab experiments

JEL classification: C78, D61, D78, I20

1 Introduction

What is the right way to assign students to schools? The school assignment problem has received a great deal of attention precisely because this question has no "correct" answer. The reason for this is that the three main design objectives of fairness, efficiency, and strategic simplicity are incompatible. There is no mechanism that is fair and efficient (Balinski and Sönmez, 1999). Worse, even when a fair and efficient assignment exists, there is no strategyproof and efficient mechanism that always selects it (Kesten, 2010). Therefore, choosing an algorithm necessarily involves trade-offs among these objectives. Since each school board has its own preferences regarding these trade-offs, the role of economists is to inform a board as to what objectives are achievable.

We introduce a new matching mechanism that is a hybrid of the two most common mechanisms. The Boston Mechanism (BM) is the most commonly used mechanism in the field, while the Deferred Acceptance algorithm (DA) is typically favored by economists.[1] Our new mechanism, the Secure Boston Mechanism (sBM), is an intuitive modification of BM that secures any school s a student has one of the top q_s priority where q_s is school s capacity, but otherwise prioritizes a student at a school based upon how she ranks it. The main advantage of BM is that it is Pareto efficient with respect to *reported* preferences, which implies that it will assign more students to their reported first choice than DA. In particular, it maximizes the first choice assignment. This feature of BM has played an important role in the policy discussions of student assignment and media reports of the outcomes of assignment procedures often focus on the fraction of students assigned to their top choice or one of their top three choices.[2] BM is also very easy for school districts to explain and very easy for parents and students to understand.[3]

However, BM is neither strategyproof nor fair, and the types of manipulations that give students an advantage under BM are easy for strategic students to identify. We present a theoretical analysis that compares our new mechanism to BM and find that, relative to BM, sBM is less vulnerable

[1] Pathak and Sönmez (2008) emphasize the disadvantages of BM and state that: "It is remarkable that such a flawed mechanism is so widely used."

[2] Vaznis (2014) provides one example of a media report that focuses on the fraction of students assigned to one of their top three choices. Educational policy professionals have also advocated for maximizing the fraction of students who are assigned to one of their top choices. See the discussions in Cookson (1995) and Glenn (1991).

[3] US school districts that use BM include Cambridge, MA; Charlotte-Mecklenburg, NC; Denver, CO; Miami-Dade, FL; Minneapolis, MN; and Tampa-St. Petersburg, FL. Strikingly, the Seattle school district replaced BM with a strategyproof mechanism only to reinstitute BM in 2011 (Kojima and Ünver, 2014).

to manipulation and more fair.[4] Further, neither BM nor sBM Pareto dominates the other. We test these theoretical predictions in a laboratory experiment. The design of the experiment was inspired by the application website used by the Wake County Public School System, which is the 16th largest school system in the United States (WCPSS, 2015). In this field setting, students are shown the number of other students who ranked a given school ranked first (Dur, Hammond, and Morrill, 2017). Borrowing this approach for transmitting information to students in the application process, we believe that our design is a highly appealing way to increase saliency in school-choice lab experiments. The experimental results suggest that, relative to BM, sBM is more strategyproof, more fair, and slightly more Pareto efficient. The effect sizes strongly support the use of sBM over BM: truth-telling is 72.9% higher with sBM, fairness is 64.7% higher with sBM, and Pareto efficiency is 3.2% higher with sBM. Further, our novel experimental design offers several methodological advantages over the designs commonly used in the experimental matching literature.

Despite the empirical prevalence of BM, economists frequently emphasize the advantages of DA, specifically that it is strategyproof, fair, and Pareto dominates any other fair assignment. Even though DA is not efficient, Kesten (2010) demonstrates that no strategyproof mechanism can Pareto dominate it.[5] Our question is whether a mechanism that is neither strategyproof nor fair can improve upon DA in terms of efficiency in equilibrium with weakly undominated strategies. Given the importance of making efficient assignments, asking whether sBM can improve efficiency relative to DA is a crucially important question with implications in school choice and beyond. We present a theoretical analysis that compares sBM to DA and find that sBM *can* Pareto dominate DA in equilibrium with weakly undominated strategies but the efficiency comparison of sBM and DA is theoretically ambiguous. We present simulation evidence that suggests that sBM often *does* Pareto dominate DA when DA is inefficient, while sBM and DA very often overlap when DA is efficient. Our results suggest that sBM should be considered as a viable alternative to DA.

Our approach to comparing sBM to the leading alternatives is as follows. Section 2 presents a theoretical analysis of sBM: (1) a comparison of sBM to BM in terms of strategyproofness, fairness, and efficiency and (2) a comparison of sBM to DA in terms of efficiency. Section 3 lays out the design

[4] In earlier studies, Pathak and Sönmez (2013) and Chen and Kesten (2014) compare manipulable and unfair mechanisms in terms of vulnerability to manipulation and level of fairness.

[5] However, using data from Cambridge, MA, Agarwal and Somaini (2014) estimate that the welfare of the average student would be lower under DA than under BM.

of an experiment that we use to compare sBM to BM in a realistic empirical setting, where the experimental results are found in Section 4. Next, Section 5 presents simulations in an environment that closely matches our experimental design to compare sBM to DA in terms of efficiency. The comparison to DA uses a simulation, and not an experiment, because we want to present the most favorable case for DA, and observe the performance of sBM relative to DA under this most favorable case. Specifically, in our simulations, students' preferences are reported truthfully under DA, which is the situation in which DA will have the best efficiency performance when students play weakly undominated strategies. In contrast, if we present experimental results on DA, the experimental literature that has followed Chen and Sönmez (2006) has provided empirical evidence that truth-telling is the norm in DA experiments but truth-telling rates remain well below 100%. As a result, we compare sBM to DA in a simulation that ignores the potential behavioral biases one might observe with DA in the lab.

Note that, sBM is a obtained by a simple, but intuitive, modification of BM. This small modification allows sBM to inherit BM's simplicity but improve with respect to BM's central flaws in that sBM is less vulnerable to manipulations and more fair. Overall, our experimental results strongly support the use of sBM over BM and our simulation results suggest that sBM should be considered as a viable alternative to DA.

Before beginning our analysis, we briefly mention several related papers that present experimental evidence on the performance of several school-choice mechanisms.

1.1 Review of the Related Experimental Literature

While there is a large theoretical literature on matching, many recent papers have used experiments to consider the relative advantages of alternative mechanisms. Many school-choice experiments provide evidence that BM has meaningful costs relative to DA or other strategyproof mechanisms. For example, the experiments of Chen and Sönmez (2006) helped convince the Boston Public Schools to abandon BM in its assignment process (Pathak, 2011). Other papers with additional evidence against the use of BM include Pais and Pintér (2008) and Calsamiglia, Haeringer, and Klijn (2010). Focusing on preference intensities, Klijn, Pais, and Vorsatz (2013) find evidence that DA is more robust to changes in cardinal preferences, relative to BM. This is in contrast to results from the theoretical literature that BM might be preferable when eliciting preference intensities is important (Abdulkadiroğlu, Che, and Yasuda, 2011).

However, several other papers find reasons to support the use of BM in certain settings. First, Lien, Zheng, and Zhong (2015) compare BM to the Serial Dictatorship (SD) mechanism, arguing that BM can dominate SD when students submit their rankings before learning their priorities. Their experimental results confirm this claim, finding that BM is more efficient than SD when preference reports precede priorities. Second, Featherstone and Niederle (2014) present experiment evidence that, in environments where truth-telling is a ordinal Bayesian Nash equilibrium under BM, subjects report truthfully with BM at rates that are high and quite similar to the truth-telling rates with DA (where truth-telling is a dominant strategy). The authors interpretation is that BM can work well in settings where truth-telling is a simple strategy for students to find and play.

This is not to first paper aiming to improve well-known mechanisms with small modifications. For instance, Miralles (2008), Dur (2013) and Mennle and Seuken (2015) propose a simple modification to BM that allows students to skip schools without available seats. Although this modification aims to decrease the level of manipulation under BM, this modified BM is not less manipulable than BM based on the comparison notion introduced by Pathak and Sönmez (2013). Kesten (2010) introduced the Efficiency Adjusted Deferred Acceptance Mechanism (EADAM) in order to improve the welfare of students. Under EADAM, students are asked to voluntarily consent to priority violations and EADAM can improve welfare only if certain students consent. Unlike DA, EADAM is manipulable. Moreover, Dur and Morrill (2016) shows that students can be harmed when they consent in the equilibrium and therefore, in equilibrium, the expected welfare gain may not be observed.

We now present our theoretical model.

2 Theoretical Model and Results

In this section, we first provide our theoretical model. Then, we define the mechanisms and give our theoretical results.

We consider a classic *school choice problem* (Abdulkadiroğlu and Sönmez, 2003). Specifically, there are a number of students, each of whom is to be assigned to a seat at a school. Each student has strict preferences over all schools. Each school has a strict priority ranking over all students.

Formally, a school choice problem consists of:

1. a finite set of students $I = \{i, j, k, \ldots\}$,
2. a finite set of schools $S = \{a, b, c, \ldots\}$,
3. a capacity vector $q =< q_a | a \in S >$,
4. a list of strict student preferences $P_I = \{P_i, P_j, P_k, , \ldots\}$,
5. a list of strict school priorities $\succ = \{\succ_a, \succ_b, \succ_c, \ldots\}$.

$\emptyset$ represents a student being unassigned, and $q_\emptyset = \infty$. Let R_i denote the associated at least as good as relation of student i. Throughout paper, we fix I, S, q, $\succ$ and we represent a problem with P. A **matching** is a function $\mu : I \to S \cup \{\emptyset\}$ such that for each $a \in S$, $|\{i \in I | \mu(i) = a\}| \leq q_a$. A matching is **Pareto efficient** if there does not exist another assignment ν such that $\nu(i) R_i \mu(i)$ for every $i \in I$ and $\nu(i) P_i \mu(i)$ for some i. A matching μ is **individually rational** if $\mu(i) R_i \emptyset$ for all $i \in I$. A matching μ is **non-wasteful** if there does not exist a student-school pair (i, a) such that $|\mu^{-1}(a)| < q_a$ and $a P_i \mu(i)$. A matching μ is **fair** if there does not exist a student-school pair (i, a) such that $a P_i \mu(i)$ and $i \succ_a j$ for some j such that $\mu(j) = a$.[6] We say a matching is **stable** if it is individually rational, non-wasteful and fair.[7]

An **assignment mechanism** ϕ is a function from a preference profile of students and priority profile of schools to a matching. Denote the outcome selected by mechanism ϕ in problem P by $\phi[P]$ and denote the match of student $i \in I$ by $\phi[P](i)$.

A mechanism ϕ is non-wasteful (Pareto efficient) if it selects a non-wasteful (Pareto efficient) outcome in any problem. Similarly, a mechanism ϕ is individually rational (fair) if it selects individually rational (fair) outcome in any problem.

A mechanism ϕ is **strategyproof** if reporting true preferences is each student's weakly dominant strategy. That is, a mechanism ϕ is strategyproof if there does not exist a student i and preference profile P' such that $\phi[P', P_{-i}](i) P_i \phi[P](i)$.

Given a school a with quota q_a, let G_a be the q_a-highest-ranked students at a. We say a student in G_a is **guaranteed** a spot at a. Note that we use the term guaranteed for expositional convenience

[6]Fairness is also known as elimination of justified envy.

[7]This is closely related to stability in the classic two-sided matching market (Gale and Shapley, 1962).

and it should be interpreted in the normative sense of "should be guaranteed." In particular, it is defined independently of any assignment mechanism, and consequently does not imply that a student is actually guaranteed assignment to that school by a specific mechanism.

2.1 Mechanism Definitions

The student proposing version of the Deferred Acceptance algorithm (DA) is defined as follows. In the first round, each student proposes to her most preferred school. Each school tentatively accepts students up to its capacity and rejects the lowest priority students beyond its capacity. In every subsequent round, each student proposes to her most preferred school that has not already rejected her. Each school tentatively accepts the highest priority students up to its capacity and rejects all others. The algorithm terminates when there are no new proposals and tentative assignments are made final. Roth and Sotomayor (1990) is an excellent resource for the properties of DA.

BM is designed as follows. In Round 1, only the first choices of students are considered. For each school, consider the students who have listed it as their first choice and assign seats at the school to these students one at a time following their priority order until there are no seats left or there is no students left who has listed it as her first choice. In Round k, repeat as in Round 1 except now a school with available seats only considers unassigned students who have ranked that school k^{th}. The procedure terminates when a student is assigned to each seat.

Note that BM is equivalent to running DA on a modified set of school priorities.[8] Specifically, given student preferences P and school priorities $\succ$, construct a modified set of priorities $\tilde{\succ}$ lexicographically based first on how the student ranks the school and second based on the student's priority at the school. Specifically, let $r_i(a)$ be the rank of school a in student i's preference list, P_i, and define $\tilde{\succ}$ as follows. For any students i and j and any school a:

- If $r_i(a) < r_j(a)$, then $i \tilde{\succ}_a j$
- If $r_i(a) = r_j(a)$ and $i \succ_a j$, then $i \tilde{\succ}_a j$

BM is equivalent to running DA using the modified priorities $\tilde{\succ}$.

[8] This way of defining BM was introduced by Ergin and Sönmez (2006).

sBM is designed as a hybrid between DA and BM. For a given school a, we fix the priority at a of the students initially guaranteed a. For all other students, we modify their priority at a in the same manner as in BM. Specifically, we first consider how the student ranks a, and we next consider the students initial priority at a. More formally, let $r_i(a)$ be the rank of school a in student i's preference list, and define $\tilde{\succ}$ as follows. For any students i and j and any school a:

- If $i \in G_a$ and $i \succ_a j$, then $i \tilde{\succ}_a j$
- If $i \notin G_a$ and $j \notin G_a$, then:
  - If $r_i(a) < r_j(a)$, then $i \tilde{\succ}_a j$
  - If $r_i(a) = r_j(a)$ and $i \succ_a j$, then $i \tilde{\succ}_a j$

Note that sBM can be defined more generally than this. Consider an arbitrary vector of integers $k =< k_1, k_2, \ldots, k_{|S|} >$ where $0 \leq k_a \leq |I|$. Given priorities $\{\succ_a\}$, define a new set of priorities $\{\overline{\succ_a}\}$ by leaving the first k_a priorities fixed and rearranging the remaining priorities according to how the students rank a. We define the generalized secure Boston Mechanism for vector k, $sBM(k)$, to be the assignment made by running DA on $\overline{\succ_a}$. Note that this defines a hybrid between BM and DA. In particular, $sBM(\overline{0}) = BM$ while $sBM(\overline{N}) = DA$.

The following example illustrates the differences between DA, BM, and sBM.

Example 1. *Suppose there are five students $\{i, j, k, l, m\}$ and four schools $\{a, b, c, d\}$. School a has the capacity for two students. Each other school has a capacity of one. Suppose the student preferences, P, and the school priorities, $\succ$ are as follows:*

i	j	k	l	m	a	b	c	d
b	c	a	b	a	i	j	i	i
c	a	b	a	b	k	l	j	j
a	b	c	c	c	j	i	k	k
d	d	d	d	d	l	k	l	l
					m	m	m	m

DA uses the priorities $\succ$. BM and sBM modify the priorities as follows, where the shaded students are guaranteed:

$\succ^{DA}$				$\succ^{BM}$				$\succ^{sBM}$			
a	b	c	d	a	b	c	d	a	b	c	d
i	j	i	i	k	l	j	i	i	j	i	i
k	l	j	j	m	i	i	j	k	l	j	j
j	i	k	k	j	k	k	k	m	i	k	k
l	k	l	l	l	m	l	l	j	k	l	l
m	m	m	m	i	j	m	m	l	m	m	m

For example, consider school a. Under BM, we first consider which student ranks a the highest, then we consider a student's priority at a. Both k and m rank a first, but $k \succ_a m$. Therefore, $\succ_a^{BM}$ gives k the highest priority and m the second highest. j and l rank a second and $j \succ_a l$. Therefore, $k \succ_a^{BM} m \succ_a^{BM} j \succ_a^{BM} l$. Finally, i is given lowest priority at a since i ranked a the lowest. In contrast, sBM protects the students initially guaranteed a. Since a has a capacity of two, the two highest priorities at a remain the same under DA and sBM. Of the remaining students, m ranks a the highest. j and l rank a the same, but $j \succ_a l$. Therefore, $i \succ_a^{sBM} k \succ_a^{sBM} m \succ_a^{sBM} j \succ_a^{sBM} l$.

The first rounds of DA, BM, and sBM are all the same. School b receives a proposal from i and l but only has capacity for one student. In all three mechanisms, student i is rejected by school b. Student i next proposes to c. Under BM, i has lost her priority at c to j and is rejected. However, since $i \in G_c$, under sBM j is rejected. j is also rejected under DA. Under BM, i is rejected by all schools until she eventually applies to d. Under both DA and sBM, in the third round j applies to a. Under DA, m is rejected and j is tentatively accepted. Under sBM, since j is not guaranteed a and m ranks a higher than j, j is rejected and m is tentatively accepted. The final assignment for the three algorithms are:

$$\mu^{DA} = \begin{pmatrix} i & j & k & l & m \\ c & a & a & b & d \end{pmatrix} \qquad \mu^{BM} = \begin{pmatrix} i & j & k & l & m \\ d & c & a & b & a \end{pmatrix} \qquad \mu^{sBM} = \begin{pmatrix} i & j & k & l & m \\ c & b & a & d & a \end{pmatrix}$$

2.2 Nash Equilibria Under the Secure Boston Mechanism

sBM induces a preference revelation game for the involved students and parents. Here, we provide an equilibrium analysis of sBM similar to the analysis provided by Ergin and Sönmez

(2006) for BM. In particular, we consider a preference revelation game under sBM. The strategies of the students are preferences over schools and being unassigned; the outcome is determined by sBM. As in Ergin and Sönmez (2006), we consider the game under complete information. Working under complete information is very common in matching theory (e.g., Pathak and Sönmez (2008) and Haeringer and Klijn (2009)).

We demonstrate our equilibrium result in the following proposition.

Proposition 1. *(i) For every problem, every stable matching under students' true preferences is a Nash Equilibrium outcome of the preference revelation game associated with sBM under complete information.*[9]

(ii) There exist problems where the Nash equilibrium outcomes, induced by weakly undominated strategies, of this game Pareto dominate (with respect to true preferences) any stable matching under the true preferences.

Proof. (i) Consider a problem P. Let μ be an individually rational, fair, and non-wasteful matching under problem P. Consider a preference profile $Q = (Q_1, ..., Q_n)$ where each student i ranks school $\mu(i)$ as her top choice under her stated preferences Q_i. If we apply sBM to problem Q, then in the first step for each school a, the number of students applying to a cannot exceed q_a and sBM terminates in the first step. That is, under the preference profile Q, sBM assigns each student to a seat at her first choice based on the stated preferences. Hence, $sBM[Q] = \mu$.

Next, we show that Q is a Nash equilibrium profile. Since μ is individually rational, none of the students can be better off by deviating and being unassigned. Now consider a student i and a school a such that student i prefers school a to $\mu(i)$. Since μ is non-wasteful and fair, all seats at school a are filled under μ and each student who is assigned a seat at school a under μ has higher priority than student i for school a. Moreover, each such student $j \in \mu^{-1}(a)$ ranks school a as her first choice under Q_j. This implies that there does not exist another strategy profile Q'_i such that i can have higher priority than any $j \in \mu^{-1}(a)$ under the school s's priority order constructed for the sBM in problem (Q'_i, Q_{-i}). Therefore, given Q_{-i}, there is no way student i can be assigned to school a. Therefore, Q is a Nash equilibrium strategy profile and μ is a Nash equilibrium outcome.

[9] One can think that this result follows from Ergin and Sönmez (2006). Although we use a similar proof technique, we present a formal proof for sBM since sBM is different than the mechanism studied by Ergin and Sönmez (2006).

Hence, any individually rational, non-wasteful, and fair matching under P is a Nash equilibrium outcome the preference revelation game associated with sBM.

(ii) We prove this part with an example.

Example 2. *Suppose there are four students $\{i, j, k, l\}$ and three schools $\{a, b, c\}$. Each school has the capacity for one student. Suppose the student preferences and school priorities are as follows, where the guaranteed students are the top priority student at each school.*

i	j	k	l	a	b	c
a	b	a	c	j	i	i
b	a	c	a	k	k	k
c	c	b	b	i	l	l
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	l	j	j

The assignment made by DA is:

$$\mu = \begin{pmatrix} i & j & k & l \\ b & a & c & \emptyset \end{pmatrix}$$

Matching μ is the student-optimal fair matching, which Pareto dominates any other fair, individually rational, and non-wasteful matching.

Notice that ~~in equilibrium~~, if a player is guaranteed her second favorite school, then it is a dominant strategy for her to submit truthful preferences. Therefore, i and j submit their true rankings in equilibrium. As a result, k is never assigned to a; either i or j will always be assigned to a. If k ranks a first, then she will be unassigned. However, if she ranks c first, then she will be assigned to c. In particular, if k ranks a school other than c as first choice, then she will be unassigned since l will get c by ranking it first. Therefore, in equilibrium k ranks c first and sBM makes the following assignment:

$$\nu = \begin{pmatrix} i & j & k & l \\ a & b & c & \emptyset \end{pmatrix}$$

Note that ν Pareto dominates μ. Hence, ν Pareto dominates any stable matching.

□

Proposition 1 shows that the set of stable matchings is a subset of the Nash equilibrium outcomes of the preference revelation game under sBM. Part (i) of Proposition 1 is also true for BM and DA. However, part (ii) does not hold under neither BM nor DA. That is, we cannot have an equilibrium outcome induced by weakly undominated strategies under the preference revelation game of DA or BM which Pareto dominates any stable matching.

Finally, we demonstrate that whenever DA is Pareto inefficient, then there exists an assignment problem with more students and where DA makes the same (inefficient) assignment but an equilibrium in undominated strategies of sBM is Pareto efficient and Pareto dominates the DA assignment.

The reason we need a larger assignment problem is to ensure that there is sufficient competition for admission. For example, consider what happens when we remove student l from Example 2.[10]

Example 3. *Suppose there are three students $\{i, j, k\}$ and three schools $\{a, b, c\}$. Each school has the capacity for one student. Suppose the student preferences and the school priorities are as follows.*

i	j	k	a	b	c
a	b	a	j	i	i
b	a	c	k	k	k
c	c	b	i	j	j
$\emptyset$	$\emptyset$	$\emptyset$			

In any equilibrium outcome, i and j are assigned to a school no worse than b and a, respectively. This follows from the fact that school a and b are guaranteed schools for j and i, respectively. Since k considers c acceptable, then c will be match of k in any equilibrium outcome. Therefore, the unique equilibrium assignment of sBM that is induced by weakly undominated strategies is:

$$\begin{pmatrix} i & j & k \\ b & a & c \end{pmatrix}$$

[10] Examples 2 and 3 are modifications of a classic example from Abdulkadiroğlu and Sönmez (2003).

This is equivalent to the DA assignment.

By removing student l from Example 2, we have changed k's incentives in equilibrium. In Example 3, k has no incentive to change her preferences as she faces no competition for school c. Therefore, we must add a student so that k faces competition for her assignment.

Theorem 1. *Consider any assignment problem $\Gamma = (I, S, P, \succ)$ such that $DA(\Gamma)$ is Pareto inefficient and let ν be any Pareto-efficient, Pareto-improvement of the DA assignment. Then their exists a Nash equilibrium (in undominated strategies) of a larger assignment problem $\Gamma' = (A', S', P', \succ')$, with $A \subseteq A'$, $S \subseteq S'$ and $P_i = P'_i$ for every $i \in A$, such that:*

- $DA_i(\Gamma) = DA_i(\Gamma')$ *for every* $i \in A$;
- $sBM_i(\Gamma') = \nu(i)$ *for every* $i \in A$.[11]

We defer the proof of this result, and other proofs, to Appendix A.

The intuition is similar to how Kesten's Efficiency Adjusted Deferred Acceptance Mechanism (EADAM) Pareto improves DA (Kesten, 2010). There, he identifies what he refers to as *interrupter* students. This is a student i that in DA applies to a school a, and has high enough priority at a that she is temporarily assigned to a and causes another student j to be rejected by a. However, j's subsequent application to other schools initiates a chain of applications and rejections that eventually results in i being rejected by a in place of a student with even higher priority. When DA is inefficient, there exists one or more interrupter students. We cannot Pareto improve DA or else an interrupter student will have justified envy. However, such an objection is "petty" in the sense that it does not improve the interrupter's assignment but only keeps other students from improving their assignment. Under DA, there is no cost for an interrupter to make such an objection. Under sBM, it is costly to rank a school highly as it lowers your priority at other schools.

In particular, suppose i is an interrupter at school a. Under DA and sBM, there is no benefit for her to rank a highly because she will not be assigned to a. Under DA, there is no cost to her to rank a highly as DA is strategyproof. However, under sBM, it is costly to highly rank a school that you will not be assigned to because this lowers your priority at other schools. Therefore, in all cases where (1) the interrupter i is not initially guaranteed her assignment under DA and (2)

[11] Note that, we do not impose any restriction on P' and $\succ'$.

she faces competition for her DA assignment, then under sBM she will not rank a above her DA assignment. Doing so would cause her to lose her DA assignment. Since she no longer "interrupts" the assignment, the sBM assignment Pareto improves the DA assignment.

2.3 Boston Mechanism vs. Secure Boston Mechanism

In this section, we compare sBM and BM based on their performance in terms of strategy-proofness, fairness, and efficiency. First, we compare incentives to manipulate preferences under sBM versus BM. sBM eliminates some incentives to manipulate under BM. For instance, under BM student might not rank a popular school at the top of their preference list and replace it with a guaranteed school because otherwise they might lose their high priority in their guaranteed school and end up in a worse school than their guaranteed school. Since sBM guarantees that a student will not be assigned worse than any of her guaranteed schools independent of her stated preferences, this kind of preference manipulation will not be seen under sBM. Moreover, for any student who is ranking a guaranteed school as her second choice in her true preferences it is weakly dominant strategy to report her true preference order under sBM.

BM is not strategyproof (Abdulkadiroğlu and Sönmez, 2003). As can be seen from Example 1, student j can be better off by ranking a as first choice under sBM. Hence, sBM is not strategyproof. Pathak and Sönmez (2013) introduce a notion which enables us to compare two mechanisms that are not strategyproof in terms of their vulnerability to manipulation. Based on their notion, mechanism ψ is *more manipulable than* mechanism ϕ if, whenever ϕ can be manipulated by student $i \in I$ in problem P, then there is at least one student $j \in I$ who can manipulate ψ in the same problem and there exists at least one problem in which none of the students can manipulate ϕ but at least one student can manipulate ψ. In Theorem 2, we show that BM is more manipulable than sBM.

Theorem 2. *BM is more manipulable than sBM.*

Next, we compare BM and sBM based on fairness. In Example 1, neither BM nor sBM selects a fair matching. That is, both mechanisms are unfair. Chen and Kesten (2014) introduce a notion to compare two unfair mechanisms based on priority violations. Specifically, mechanism ψ is *more fair than* mechanism ϕ if whenever ϕ can select a fair matching in problem P then so does ψ in the same problem and there exists at least one problem in which ψ selects a fair matching but ϕ does not. In the following proposition we show that sBM is superior to BM based on this definition.

Proposition 2. *sBM is more fair than BM.*

Although sBM has better features than BM, it is not Pareto efficient. For instance, in Example 3 under the true preferences sBM assigns i to a and j to b and they prefer to swap their assignments. However, our final result considers whether BM Pareto dominates sBM.

Proposition 3. *BM does not Pareto dominate sBM.*

If we extend the set of priorities that have to be respected, can we further decrease the level of gaming under sBM? The following example illustrates that, surprisingly, the answer is no.[12]

Example 4. *Suppose there are five students $I = \{i, j, k, l, m\}$ and five schools $S = \{a, b, c, d, e\}$. Each school has the capacity for one student. Suppose that student preferences and the school priorities are as follows.*

i	j	k	l	m	a	b	c	d	e
a	b	a	c	a	i	j	k	i	i
b	a	b	d	c	j	k	m	j	k
c	c	d	a	e	k	i	l	l	j
d	e	e	e	d	l	l	i	k	l
$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	m	m	j	m	m

Under this problem, sBM selects the following matching:

$$\mu = \begin{pmatrix} i & j & k & l & m \\ a & b & d & c & e \end{pmatrix}$$

Note that, under μ students i, j, and l are assigned to their top choices. Hence, they do not have an incentive to misreport when sBM is applied to this problem. Moreover, k cannot get either a or b

[12] It is worth mentioning that we can compare two different versions of sBM mechanism defined under two different guaranteed set of students G' and G'' such that for each $a \in S$ $G'_a \subseteq G''_a$ and G''_a is composed of the q-highest-ranked students at a where $q \leq q_a$ in terms of vulnerability to manipulation and fairness. In particular, sBM defined under G' is more manipulable than sBM defined under G''. Moreover, sBM defined under G'' is more fair than sBM defined under G'. These results can be shown by following the steps of Theorem 2 and Proposition 3, respectively.

by misreporting because the students assigned to these schools have the highest priorities. Similarly, m cannot get a by misreporting. Note that, m cannot get c by ranking it as her second or lower choice. Finally, if m reports c as her top choice, then she starts a rejection chain that result in her rejection from c due to k's application. Hence, none of the students can manipulate sBM under this problem.

Now consider a version of sBM in which for each $s \in S$, G_s *includes the students with highest* $q_s + 1$ *priority. That is,* $G_a = G_d = \{i, j\}$, $G_b = \{j, k\}$, $G_c = \{k, m\}$, *and* $G_e = \{i, k\}$. *When we run this version of the sBM mechanism, the following matching is selected:*

$$\nu = \begin{pmatrix} i & j & k & l & m \\ a & b & c & d & e \end{pmatrix}.$$

Here, if k reports d as the top choice, then this version of the sBM will select the following matching:

$$\nu' = \begin{pmatrix} i & j & k & l & m \\ a & b & d & e & c \end{pmatrix}.$$

Hence, k can manipulate this version of sBM with an extended set of priorities that need to be respected.

To understand how sBM compares to BM in practice, we conduct an experimental analysis in the laboratory.

3 Experimental Design

3.1 Robotic Subjects Design

In the experiment, subjects submit a preference ranking as one of several students seeking assignment. There are two key features of our design. First, each subject is in a group where all other students are played by the computer. Second, subjects are given neither the complete preference ordering of all students (complete information) nor distributional information of the preferences of all students (incomplete information). Instead, the experimental interface tells subjects the number of other students who rank each school first in their ranking. A key advantage of an approach based on these two features is that we do not have to provide subjects with any information specifically about the preferences of the robotic subjects or specifically about the strategies of the robotic subjects. Instead, subjects are given information about the outcomes of the robotic subjects' ranking behavior, which embeds information about preferences and strategies.[13]

A first alternative is to tell subjects that robotic subjects will always rank truthfully and give subjects information about the distribution of preferences for robotic subjects. One disadvantage of this approach is that it provides subjects with a focal strategy of truth-telling that may bias their behavior. A second alternative is to tell subjects that robotic subjects will rank following the behavior of human subjects in previous experiments (i.e., empirical robots). Chen, Jiang, Kesten, Robin, and Zhu (2015) is the first paper to take this approach in the school choice literature but it has been used in other types of experiments. While empirical robots provide an interesting comparison for our design, there are several advantages of our approach. First, the empirical-robots protocol requires the researcher to have previous all-human experiments that match the exact design, which is a non-trivial administrative challenge in conducting experiments.

Second, we argue that our approach is easier for subjects to understand, where playing against empirical robots requires subjects to contemplate the ranking behavior of previous human subjects and understand the procedure used to "train" robotic subjects. In contrast, a subject in our experiment simply has to understand that their robotic opponents ranked in a way that generated the first choices they are shown on their screen. In Figure 1, the "# of Students' Top Choice" column shows the robots first-choice rankings; we discuss the experimental interface more fully in the next subsection.

Third, relative to an empirical robots design, our approach has less strategic uncertainty because the exact first-choice rankings of all robots are shown to the human subjects. An additional reduction in strategic uncertainty gives us a cleaner look at the strategic decision making of our

[13] It is certainly the case that students in the field are competing against other students, not robots. However, many school choice settings involve thousands of students. This implies that a single student cannot contemplate the strategic behavior of her opponents at the individual level and instead must respond to (her perception of) their aggregate behavior. This is similar to the strategic setting in a human-robots design. In fact, several recent theoretical papers in matching have used a large-market approximation when considering the strategic behavior of an individual student, which again is similar to our human-robots design (e.g., Kojima, Pathak, and Roth (2013)).

human subjects in isolation from their beliefs about the strategic play of past human opponents. Fourth, as pointed out by Chen, Jiang, Kesten, Robin, and Zhu (2015), a human-robot design provides the researcher with subject observations that are statistically independent because they are no interactions between human subjects.

Importantly, our design was inspired by the application website used by the Wake County Public School System, which is the 16th largest school system in the United States (WCPSS, 2015). In the screenshot in Figure 2, a student ranking schools in this field setting is shown the number of "Current 1st Choice Applicants," which is the number of other students who currently have a given school ranked first (Dur, Hammond, and Morrill, 2017). Borrowing this approach for transmitting information to students in the application process, we believe that our design is a highly appealing way to increase saliency in school-choice lab experiments.

3.2 Experimental Sessions

Using this design, the experiment was run using zTree (Fischbacher, 2007) and conducted at the experimental lab at North Carolina State University during the Spring of 2015. 81 subjects participated in the experiment, which lasted around 90 minutes. Because our design requires no interactions between human subjects, subjects were asked to arrive during a 30 minute window and each subject began their experiment as soon as they arrived and left as soon as they were paid in private. We use a two-by-two-by-two design, with treatment variation in mechanism (BM or sBM), market size (four schools or eight schools), and knowledge concerning one's tie-breaking lottery number (known or unknown). Subjects participated in only one mechanism (between-subjects design) but in periods of both markets sizes (within-subjects design) and both information settings (within-subjects design).

Subjects participated in 32 periods, where the size of the market varied with an ABBA/BAAB design (e.g., an example of such an order is 8 periods of four schools, followed by 8 periods of eight schools, 8 periods of eight schools, and finally 8 periods of four schools). In periods with four schools, subjects were told that there were 11 students played by the computer and each school had a capacity of three. In periods with eight schools, subjects were told that there were 239 students played by the computer and each school had a capacity of thirty. Finally, each subject participated in periods of both lottery settings with an AB/BA design (i.e., half of the subjects were shown their lottery number in the first 16 periods but not the last 16 periods, while the other half of subjects saw the reverse order). The treatments were run within the same sessions, mixed in a way that roughly balanced each treatment for each of the four sessions.

To further understand the design, see the ranking screen of a four-school, known-lottery-number period shown in Figure 1. The subject sees that her favorite schools in descending order are C, B, A (her district school), and D. The subject also sees that seven robotic subjects ranked her favorite school first, while all four remaining robotic subjects ranked her district school (A) first. Given that schools in the four-school environment have a capacity of three, this example exhibits strong demand for both the subject's favorite school and her district school. The subject in this example is shown her lottery number, while the interface simply removes this information (but is otherwise the same) in unknown-lottery-number periods. The interface is exactly the same for BM and sBM sessions, only the background code that runs the allocation is different.

At the beginning of each session, subjects read the instructions, during which time they were given headphones and directed (via the zTree interface) to watch a video discussing the instructions. The video was narrated by one of the authors and neither the instructions nor the video guided subjects toward particular strategies, instead only providing information about the mechanism (either BM or sBM) and the rules of the game. The instructions are in Appendix B, which also contains links to the instructional videos. The zTree interface did not allow subjects to continue until the time on the video was reached and subjects were free to rewind the video if they wished.

Following the instructions, subjects took an incentivized quiz that included an exercise in determining the allocation of the mechanism (either BM or sBM) using an example with four students and four schools. Our quiz and the example were adapted from Chen and Kesten (2014). The first four questions asked subjects to determine the allocation of each student in the example. Then, subjects were directed to watch a second video that explained the correct answers to the four allocation questions and review the instructions regarding the mechanism within the context of the example. The final part of the quiz involved an additional nine questions, followed by a third and final video reviewing the answers and encouraging subjects to raise their hands with any final questions before the experiment began.[14] Subjects were told that they must repeat the quiz (with no earnings on the retake) if they answer less than 10 questions correct on the quiz. The

[14] Again, see Appendix B for links to the instructional videos.

average subject answered 10.48 questions correctly, and two-thirds of subjects answered at least the required 10 questions correct, while 43% of subjects missed zero or one question.

Before the experiment began, subjects were asked to enter their student ID number. In the recruitment, subjects were told that they would provide their student ID number and that the experimental data would be matched to data from their registration and enrollment records in order to see whether "such things as GPA, academic major, etc. can enhance the predictive power of standard economic models typically used to analyze the data." Next, the experiment began, around 45 minutes after the subject arrived. As explained earlier, subjects participated in 32 experimental periods. After the experiment ended, subjects participated in an incentivized elicitation of their risk and ambiguity preferences, following the multiple-price list approach of Holt and Laury (2002).

The risk elicitation came first for all subjects and presented a list of 10 paired choices between a 50/50 lottery and a sure payoff whose value varied around the expected value of the lottery. Finally, the ambiguity elicitation presented subjects the same list of choices with the same values for the sure payoff, where the only change is that the lottery had uncertain probabilities. Risk averse subjects require a lower sure payoff to switch from the lottery, while ambiguity averse subjects switch from the lottery to the sure payoff at a lower value of the sure payoff relative to the same subject's switching point in the risk elicitation. Subjects made 10 choices for each elicitation (20 in total) and one was chosen at random for payment for each elicitation (two in total).

Payoffs were expressed to subjects as points and they were told that every two points was worth $1. Earnings in dollars were $26.91 on average, with a range of $14.75 and $36.75. These numbers include a $5 show-up fee and payments from the quiz and each elicitation. Earnings from the experiment itself were $14.23 on average, with a range of $4.00 and $20.00. Earnings from the quiz were $2.62 on average, relative to a maximum possible quiz earnings of $3.25. Finally, earnings from the risk and ambiguity elicitations were $2.55 and $2.50 on average, respectively.

Before defining the outcomes of interest, we discuss the payoffs for each subject at each school, that is, their preferences. All subjects (the human subject and the robotic subjects) were assigned a randomly drawn cardinal preference for each school. Preferences for a given school were correlated across subjects such that some schools were more popular in a given period but the popular schools changed from period to period. Consistent with our robotic subjects design, the instructions told subjects: "You will be able to tell which schools are popular because you are shown the number of computer participants that rank each school first." This is another advantage of showing the number of first choices because it concisely conveys all information about preferences instead of having to separately specify which schools were popular.[15]

The human subject's cardinal preferences were always drawn such that the school with the highest payoff was valued at 20 points, the lowest payoff at 0 points, and the payoffs in between valued at uniformly distributed integers. The intermediate payoffs are different with four schools, relative to eight schools; the exact way preferences were described to subjects is in Appendix B and an example set of payoffs is shown in Figure 1. The human subject's district school was constrained to be either her second or third favorite school (with four schools) or her second, third, or fourth favorite school (with eight schools). Finally, robotic subjects' preferences were draw in the same way as with the human subject and the ordinal ranking that resulted from their cardinal preferences were submitted by each robotic subject. That is, robots had cardinal preferences that were drawn in the same correlated-preference environment as the human subjects and robots submitted their preferences truthfully. But since we did not want to convey this information to subjects, which could bias their strategic play, we instead showed the number of first choices that resulted from the robots' preference reports.

3.3 Outcomes of Interest

We now discuss our hypotheses of interest in the experiment.

Hypothesis 1. *More students will report their preferences truthfully with sBM than with BM.*

Truth-telling has been defined various ways for various mechanisms in the literature. For BM, Chen and Sönmez (2006) define truth-telling as submitting the entire preference ranking truthfully, while for DA, Chen and Sönmez (2006) define truth-telling as submitting the preference ranking truthfully up to the student's district school. Other papers have followed this definition in general (Calsamiglia, Haeringer, and Klijn, 2010; Featherstone and Niederle, 2014; Ding and Schotter, 2014; Guillen and Hing, 2014) but several papers now look at more than one measure of truth-telling for a given mechanism, which is the approach we will follow. Specifically, Hypothesis 1 will be tested using four measures as follows: $true_1$, submitting one's favorite school first; $true_d$, submitting

[15] One can deduce in Figure 1 that schools A and C are popular in this period.

the preference ranking truthfully up to the student's district school; $true_a$, submitting the entire preference ranking truthfully; and, following Chen and Sönmez (2006), $true_{cs}$, submitting the entire preference ranking truthfully for BM and submitting the preference ranking truthfully up to the student's district school for sBM.

Hypothesis 2. *The increase in truth-telling with sBM, relative to BM, will be more pronounced in those cases when the student's district school is second in her ordinal preference ordering.*

When a student's district school is her second favorite school, Section 2 shows the it is weakly dominant strategy to report truthfully with sBM, while this is not the case when the district school is ranked lower than second. With BM, truth-telling is not in general an equilibrium irrespective of the position of the student's district school. As a result, we hypothesize that the truth-telling advantage of sBM over BM will be concentrated in cases where theory predicts that reporting truthfully is an equilibrium.

Hypothesis 3. *District school bias will be lower with sBM than with BM.*

One type of non-truth-telling behavior is known as district school bias, which is when a student ranks her district school higher in her submitted ranking than in her true preference ranking (Chen and Sönmez, 2006; Calsamiglia, Haeringer, and Klijn, 2010). District school bias is measured in two ways: dsb_1, submitting one's district school first, and dsb, submitting one's district school higher than if truthful. Recall that the student's district school was never allowed to be her favorite school (such a setting is uninteresting as the dominant strategy is obvious). As such, dsb_1 involves moving the district school to the top of the submitted list, while dsb involves moving the district school up but not necessarily to the top.

Hypothesis 4. *BM will assign more students to their reported first choice, relative to sBM. In contrast, BM will not assign more students to their true first choice, relative to sBM.*

Following Chen and Kesten (2014), we will refer to these two measures as "first choice accommodation" according to reported and true preferences, respectively. It is a clear feature of BM that more students will receive the school they rank first and this metric is often reported by school districts as a measure of the success of the assignment.[16] However, it is more important to assign

[16]For one example from the Boston Public Schools, it was reported that 47.3% of students received their report top choice in 2014 (Vaznis, 2014).

students to their true top choice and we hypothesize that BM's advantage over sBM will be smaller or nonexistent with respect to true preferences.

Hypothesis 5. *sBM will assign fewer students to a school they prefer less than their district school, relative to BM.*

Recall that, in our design, the student's district school is always her second, third, or fourth favorite school. Hypothesis 5 directly follows from the fact that sBM secures a student's position at her district school.

Hypothesis 6. *There will be fewer instances of justified envy with sBM than with BM. That is, sBM assignments will be more fair than BM assignments.*

Hypothesis 6 follows directly from Proposition 2.

Hypothesis 7. *BM will not have higher levels of efficiency, relative to sBM.*

Hypothesis 7 follows directly from Proposition 3. We now present the experimental results.

4 Comparison of sBM and BM Using a Lab Experiment

To verify that our between-subjects design was appropriately randomized across subjects, Table 1 shows summary statistics for the 41 and 40 subjects who participated in BM and sBM sessions, respectively. We do not find statistically significant differences in any treatment setting or subject characteristic. Subjects were balanced across sessions and the exact same number of subjects passed the quiz in each mechanism. The next three treatment setting variables require further explanation.

First, to understand the demand at favorite variable, recall that each human subject is in a group with all other students played by robotic subjects. Subjects are shown the first choice rankings of all robotic subjects and demand at favorite is the fraction of robotic subjects that ranked the human subject's favorite school first. Second, the favorite preference intensity variable is the difference in the cardinal preference of the subject's favorite school and her second favorite school, where a larger difference implies that the subject has a more intense preference for her favorite school. Third, the district second favorite variable is a dummy variable that equals one when the subject's district school is her second favorite school and equals zero when it is her third or fourth favorite school. Each of these treatment settings are similar on average across the subjects who participated in each mechanism.[17]

For subject characteristics, our subjects have little past experiments with economics experiments, are overwhelmingly American, and primarily white students. We have a roughly equal balance of males and females; likewise, we have similar numbers of risk averse and risk neutral subjects, while the majority are ambiguity neutral. Finally, the data collected from registration and enrollment records are primarily useful in providing financial need and academic performance variables that would be otherwise difficult to reliably measure. Nearly 30% of subjects are receiving need-based financial aid. Academic performance proxies for cognitive abilities, measured by SAT score (average score is approximately 1230) and high-school GPA (average is approximately 4.3 on a 5 point scale).

In the results that follow, statistical tests are nonparametric and regression models include subject-level random effects. The regression specifications are panel-data logit models for binary outcomes and panel-data linear models for continuous outcomes. Throughout, marginal effects at the mean are shown along with heteroskedasticity-robust standard errors.

Result 1. *Truth-telling is higher with sBM, relative to BM, to a quantitatively large and statistically significant degree.*

Concerning Hypothesis 1, Tables 2 and 3 show two different approaches to compare the rates of truth-telling between BM and sBM. Table 2 provides four different measures of truth-telling. When we consider whether a subject ranked all schools according to her true preferences, $true_a$, truth-telling is 4.8 percentage points (around 38%) higher with sBM relative to BM. Using the other three definitions, the increase in truth-telling with sBM is much higher, though all effects are similar in that they are all highly statistically significant. The preferred measure in Chen and Sönmez (2006), $true_{cs}$, is a hybrid of ranking truthfully up to the district school, $true_d$, for sBM and ranking truthfully for all schools, $true_a$, for BM. Using $true_{cs}$, the increase with sBM is 15.6 percentage points, or 123%. For the remaining analysis, we define truth-telling as $true_d$ because

[17] Demand at favorite averages around 51%, which implies that around half of the robotic subjects ranked the subject's favorite school first. This is consistent with our correlated-preference environment, described earlier. Next, favorite preference intensity averages 5.7 points, where again two points are worth $1. This implies that subjects have a lot to lose by not getting their favorite school. Finally, the subject's district school is her second favorite school around 60% of the time.

its results are conservative and it is a natural way to think about truth-telling.[18]

Next, Table 3 presents a regression analysis of truth-telling, defined as ranking truthfully up to the district school, $true_d$. Similar to the results in the raw data shown in Table 2, sBM increases truth-telling by approximately 11 percentage points. The effect of sBM on truth-telling is 72.9%, which is a very large effect and strongly supports Hypothesis 1, suggesting that sBM is effective in reducing manipulations in preference reports. Demographic and other subject-level covariates are included in columns (2) and (4) but their results are shown in Table 12. The included subject characteristics are those shown in Table 1 and are discussed in their effect on several outcomes of interest after the discussion of the primary findings. Incomplete records for subject covariates, mainly for SAT scores and GPAs, lowers the sample size when demographics are included.

Further results in the table show that there is more truth-telling when there are four schools, which is intuitive because there are more manipulations when there is more to manipulate. Showing subjects their lottery number that is used to break ties does not have a large effect on truth-telling. Next, there is no trend toward or away from truth-telling as the experiment progressed but there are session effects such that the first session had lower rates of truth-telling in the final half of the experiment. We do not have an explanation for this finding. Subjects who did well on the pre-experiment comprehension quiz have a slight tendency to rank more truthfully than subjects who scored lower. Further, more competition for a slot at one's favorite school, measure by demand at favorite, increases truth-telling but only when considering the final 16 periods. A more intense cardinal preference for one's favorite school results in more truth-telling, which is intuitive. Finally, a subject is more likely to report truthfully when her district school is her second favorite school, relative to the district school being ranked third or fourth.

This leads us directly into our next result, which further analyzes the considerable increase we found in truth-telling with sBM. To look deeper, we note again that truth-telling is not, in general, an equilibrium with sBM. However, in our theoretical analysis of the Nash equilibrium of sBM, we show that truth-telling is an equilibrium with sBM when a subject's district school is second in her ordinal preference ordering. That is, when your guaranteed school is your second favorite, you can not hurt yourself with sBM by ranking truthfully.

[18]The rates of truth-telling we observe with BM are in line with the previous experimental literature. For example, Chen and Sönmez (2006) find a truth-telling rate of 13.9% with BM in a setting that is comparable to ours (the "designed environment").

Result 2. *The increase in truth-telling with sBM, relative to BM, is concentrated in those cases when the subject's district school is second in her ordinal preference ordering.*

Table 4 disaggregates the data according to the position of the subject's district school. The results strongly support Hypothesis 2. Subjects are not simply reporting truthfully more often with sBM but are instead are more likely to report truthfully when theory predicts that they should. More specifically, sBM increases truth-telling by around 17 percentage points when the district school is the subject's second favorite school but only around 3 percentage points when the district school is ranked third or fourth. We always find that truth-telling is higher with sBM but that the aggregate result is driven by those settings in which theory makes a sharp prediction of truth-telling with sBM. In other words, our experimental findings say that subjects are not only doing better with sBM, but they are doing better for the right reasons.

Result 3. *District school bias is lower with sBM, relative to BM, to a quantitatively large and statistically significant degree.*

To address Hypothesis 3, we use two definitions of district school bias: dsb_1, submitting one's district school first, and dsb, submitting one's district school higher than if truthful. Table 5 shows raw differences and Table 6 show regression results, as before. Along both measures of district school bias, the effect of sBM is more than 30 percentage points.[19] Strikingly, less than 5% of subjects rank the district school first with sBM, while more than 37% do so with BM, an effect size of 673%! Controlling for the full set of covariates in Table 6 results in effect sizes that are similar and again highly statistically significant. It is important to emphasize that addressing district school bias is crucial because this is that most prevalent bias that has been identified in previous laboratory experiments and has also been found to be important in the limited empirical evidence from the field.[20]

Other results in Table 6 show that district school bias is weakly lower when there are four schools, rather than eight schools, consistent with the truth-telling result. Differently than with truth-telling, district school bias is more common when a subject is shown her tie-breaking lottery

[19] The rates of district school bias we observe with BM are in line with the previous experimental literature. For example, Chen and Kesten (2014) find a district school bias rate of 47.8% with BM.

[20] See Dur, Hammond, and Morrill (2017) for evidence from the Wake County Public School System, which is one of the largest school systems in the United States.

number, providing one rationale for not revealing this information to students in the field.[21] There is a slight trend, over the course of the experiment, away from district school bias and, again, we find surprising session effects where subjects in the first session are less truthful in the second half of the experiment. Further, as before, subjects who demonstrated a better understanding of the experimental instructions (by passing the quiz) submit manipulated reports slightly less often.

While demand at favorite had only a weak effect on truth-telling, it strongly increases district school bias, implying that subjects are responding to competition at their favorite school and moving their district school up when such competition is strong. Finally, consistent with the truth-telling results, there is less district school bias when a subject's preferences are more intense for her favorite school results and when the district school is second in her true preference order.

Result 4. *BM assigns dramatically more subjects to their reported first choice, relative to sBM, but BM and sBM assign similar numbers of subjects to their true first choice.*

To test Hypothesis 4, Tables 7 and 8 summarize first choice accommodation with respect to reported and true choices, respectively. BM assigns 72.7% of subjects to their reported first choice, which is much higher than the 36.9% rate found with sBM.[22] In contrast, BM assigns 95.3% of subjects to one of their top three reported choices, which is statistically significantly lower than the nearly 100% rate found with sBM. More importantly, BM and sBM assign similar numbers of subjects to their true first choice, 12.7% with BM versus 12.2% with sBM. Further, sBM assigns 79.2% of subjects to one of their top three true choices, which is a statistically significant increase relative to the 74.7% rate found with BM. It is useful that sBM does better, *even with respect reported preferences*, in assigning to one of the top three choices because policymakers and school districts are commonly focused on this statistic. For example, numerous media accounts reported that Boston Public Schools assigned 73.1% of applicants to "one of their top three picks" and this statistic is sometimes cited before the statistic concerning top choices (Vaznis, 2014).

Result 5. *sBM assigns fewer subjects to a school they prefer less than their district school, relative to BM.*

[21] Pais and Pintér (2008) present experimental evidence that district school bias is higher in a more complete information environment (similar to our finding) but they also find that truth-telling is lower with more information (where we find no effect of information).

[22] The rates of first choice accommodation we observe with BM are in line with the previous experimental literature. For example, in Chen and Kesten (2014), around 60% of subjects were assigned to their reported first choice, while 12% of subjects were assigned to their reported true choice.

Table 9 shows how often subjects are assigned to a school that they prefer more than their district school (column (1)), to their district school (column (2)), and to a school that they prefer less than their district school (column (3)). The results provide strong support for Hypothesis 5. Specifically, 9.0% of subjects are assigned to a worse school with BM, which is meaningfully more than with sBM. Note however that, with sBM, a subject cannot be assigned to a school that she prefers less than her district school unless she ranks her district school lower than such a school. Playing such a strategy is weakly dominated but our results suggest that a very small number of subjects do so with sBM. However, even with these small number of mistakes with sBM, the results suggest that sBM does better than BM in terms of avoiding assignment at undesirable schools, that is, schools preferred less than one's district school.

Result 6. *There are fewer instances of justified envy with sBM, relative to BM, to a quantitatively large and statistically significant degree. That is, sBM is more fair than BM.*

Proposition 2 says that, while sBM is not fair, it is more fair than BM. Table 10 presents results that strongly support Hypothesis 6. The mean number of instances of justified envy with BM is 0.91, which suggests that the human subject has a justified claim on 0.91 seats of robotic subjects in the average BM allocation; the mean number of instances of justified envy with sBM is 0.47.[23] From the sBM coefficient in Column (2) of 0.588, the effect of sBM on fairness is 64.7%. Other results in Table 10 suggest that justified envy is lower with four schools, which is intuitive. Further, there is a trend toward more fair assignments as the experiment progresses. There are again surprising session effects in terms of fairness. Fairness is higher (less justified envy) when a subject's favorite school was more popular and fairness is also higher when the subject's district school is their second favorite school, relative to the district school being ranked third or fourth.

Result 7. *Efficiency is higher with sBM, relative to BM, with a moderate effect size.*

In Table 11, the dependent variable is the subject's payoff in points in each period, where two points are worth $1. Because only a subset of periods were chosen for payment, it is easier to analyze earnings in points. Payoffs are our measure of efficiency and are used to test Proposition

[23]Other definitions of justified envy provide similar results. The proportion of robotic subjects of whom the human subject has justified envy falls from 1.3% with BM to 0.6% with sBM. The fraction of that the time that the human subject has at least one instance of justified envy falls from 18.4% with BM to 9.0% with sBM. The effect of sBM on justified envy is similar with all three definitions and the reduction is always highly statistically significant.

3. The sBM coefficient in Column (2) of 0.456 points implies that the effect of sBM on efficiency is 3.2% . However, the effect is only statistically significant in the full specification when considering all periods. We consider this to be weak evidence that sBM is more efficient than BM but strong evidence in favor of Hypothesis 7 that BM will not Pareto dominate sBM. This is consistent with the discussion of Hypothesis 4, where sBM does slightly better in assigning subjects to one of their three top choices.

Efficiency is higher with four schools, relative to eight schools, while there is not a large difference when the subject's lottery number is shown. Further, efficiency increases over the course of the experiment. Combined with the earlier results on the period trend, we find overall that subjects learn to do a little better as the experiment progresses but that there is not a lot of evidence for substantial period effects. In contrast to the session effects found earlier, average efficiency is similar in all sessions. Further, quiz performance is associated with slightly higher levels of efficiency. Concerning treatment settings, subjects do worse in periods in which their favorite school was more popular, not surprisingly. Next, subjects do worse when they more intensely prefer their favorite school because this implies that their second favorite school has a lower payoff. Finally, subjects do better when their district school is their second favorite school.

To close this section, we note that Table 12 shows the effects of demographic and other subject covariates on four separate outcomes of interest, displayed in one table for ease of comparison. The four columns consider truth-telling, district school bias, fairness, and payoffs, respectively. For risk averse subjects, economics majors, and minority subjects, we find more truth-telling and less district school bias. The fact that risk averse subjects are more truthful runs counter to the interpretation that district school bias is driven by risk avoidance. Further, subjects who have participated in past experiments report more truthfully. Other subject characteristics (including the variables from registration and enrollment records) have effects that are quantitatively small and statistically insignificant. Covariates that are associated with higher levels of fairness (i.e., lower levels of justified envy) tend to be those that are associated with higher levels of truth-telling. Finally, few covariates have any meaningful predictive power in terms of efficiency.

5 Comparison of sBM and DA Using Simulations

In the previous sections, we compared sBM to BM in terms of strategyproofness, fairness, and efficiency. As sBM is a hybrid of DA and BM, we now compare sBM to DA. Since DA is strategyproof and fair, it is impossible for sBM to outperform DA in any instance based on fairness or vulnerability to manipulation. In this section, we use simulations to compare sBM to DA in terms of efficiency. Given the importance of making efficient assignments, asking whether sBM can improve efficiency relative to DA is a crucially important question with implications in school choice and beyond.

Our simulated environment is designed to be consistent with the experimental design, which allows us to compare the three mechanisms.[24] Specifically, there are 12 students, 12 available seats, and 4 schools (i.e., each school has 3 available seats). Each student has a secure school and her secure school is either second or third in her true preference order. The relative order of the other schools is selected randomly. Each school has a priority order in which the school's secure students have the highest priority, while the non-secure students are randomly ordered in the remaining priority slots.

For each simulation run, we solve for a Nash equilibrium strategy to determine the ranking that each student will optimally submit under sBM. Given that DA is strategyproof, the DA assignment is determined assuming truth-telling. In this environment, we ran 5,000 simulations, with new draws for priorities and true preferences. To present the results, we make Pareto comparisons between the sBM assignment and the DA assignment, separately for simulations runs where the DA assignment is Pareto efficient versus Pareto inefficient. In these results, DA makes an efficient assignment in 2740 runs (54.8% of simulation runs). Among the 2740 runs where the DA assignment is efficient, we find the following comparison:

- 2737: DA and sBM make the same assignment
- 3: assignments are different but cannot be Pareto ranked.

Among the 2260 runs where the DA assignment is inefficient, we find the following comparison:

[24] In addition to the experimental environment, we have explored a wide range of alternative scenarios. These robustness results are detailed in the Supplemental Appendix, which is available at https://goo.gl/ZQpKrQ. An overview of the robustness results is provided at the close of this section.

- 1583: DA and sBM make the same assignment
- 2: assignments are different but cannot be Pareto ranked
- 675: assignments are different and sBM strictly dominates DA.

Importantly, the sBM assignment strictly dominates the DA assignment in 13.5% of runs. Summarizing, sBM often Pareto dominates DA when DA is inefficient, while sBM and DA very often overlap when DA is efficient.

Next, we present an alternative analysis of the same simulation results. Table 13 presents the average number of students assigned to each choice in the left panel and the average number assigned to this position or higher in the right panel. Standard errors are shown in parentheses. We find that in equilibrium, sBM assigns more students to their true first choice than does DA.[25] To investigate whether these differences are meaningful, in a statistical sense, we use nonparametric medians tests.[26] sBM assigns statistically significantly more students to their first choice, statistically significantly fewer students to their second choice, and statistically significantly fewer students to their third choice. To look for a stochastic dominance relationship, the right panel shows the cumulative average number of students assigned to each choice. The results show that sBM first-order stochastically dominates DA, where the dominance relation is statistically significant.

These results provide evidence that sBM should be considered as a viable alternative to DA. Further, in the Supplemental Appendix, we explore 32 additional simulation environments, where the number of schools ranges from 3 to 16 and capacities range from 2 to 12.[27] The results show that sBM Pareto dominates DA in between 4.7% and 58.9% of runs, with an average sBM Pareto domination rate of 26.3%. This suggests that the results reported in detail above (where sBM Pareto dominated DA 13.5% of the time) are conservative. Moreover, we find that DA Pareto dominates sBM in between 0.0% and 0.3% of runs, with an average DA Pareto domination rate less than 0.1%. In words, DA Pareto dominates sBM less than one-tenth of one percent of the time,

[25] We also consider sBM assuming each student submits her true preferences in order to consider the performance of sBM with nonstrategic students. In these results, sBM (under truth-telling) does better than DA and sBM (under equilibrium play) in first-choice assignments but worse in cumulative assignments through the first two choices. These results are available from the authors upon request.

[26] We test for differences in the distributions in their central location, with no assumptions about the parametric form of the distributions. The use of nonparametric tests is consistent with the experimental literature and is generally more conservative than parametric tests.

[27] Again, the Supplemental Appendix is available at https://goo.gl/ZQpKrQ.

while sBM Pareto dominates DA around one fourth of the time.

Finally, the robustness results in the Supplemental Appendix shed light on the situations in which DA is Pareto inefficient. Specifically, we find that DA becomes more inefficient when the number of students increases, relative to the number of schools. In real world assignment problems, there are typically a large number of students, which suggests that DA is likely to result in meaningful efficiency losses in the settings that are typically of interest. The implication is that sBM will likely have a lot of scope for efficiency improvements in the field.

6 Conclusions

We introduce a modified version of the most widely used student assignment mechanism, the Boston Mechanism (BM). We present theoretical and experimental analyses of the performance of the new mechanism, the Secure Boston Mechanism (sBM). Our theoretical analysis of sBM, relative to BM, shows that BM is more manipulable than sBM, sBM is more fair than BM, but neither mechanism Pareto dominates the other in general. We then compare sBM to the Deferred Acceptance algorithm (DA), which has been promoted as a superior mechanism to BM. Theoretically, we show that sBM *can* increase efficiency relative to DA but the efficiency comparison of sBM and DA is theoretically ambiguous. Our empirical analysis proceeds in two parts: a comparison of sBM and BM in a lab experiment and a comparison of sBM and DA in a simulation. We use an experiment to compare sBM to BM because neither mechanism is strategyproof, thus we need empirical evidence on how students actually behave in these mechanism. In contrast, we use a simulation to compare sBM to DA in the environment that is most favorable to DA in order to avoid biasing the comparison in favor of sBM.

The experimental design has two key features. First, each subject plays against robotic subjects. Second, subjects are shown the number of other (robotic) subjects ranking each school first, along with their own preferences. This environment succinctly conveys information to subjects in a field-relevant way to ensure saliency. The results strongly support the use of sBM over BM, finding that, relative to BM, sBM is less vulnerable to manipulation, more fair, and slightly more efficient. The simulation environment is chosen to closely match the setting of our experiment. We find that sBM often Pareto dominates DA when DA is inefficient, while sBM and DA very often overlap when DA is efficient. We provide a large number of simulations in alternative environments that demonstrate that our simulation results are robust and the analysis in Section 5 are representative of the general pattern.

Our results have important implications for how we assign students to public school seats, an allocation problem that involves what is probably our most important public resource. The advantages of DA are well known and economists have been recently successful in convincing policymakers to move away from assigning students using BM. However, the inefficiencies associated with DA are important to acknowledge. Our theoretical and empirical results suggest that the Secure Boston Mechanism is clearly preferable relative to the Boston Mechanism and should be considered as a viable alternative to the Deferred Acceptance algorithm.

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A Omitted Proofs

A.1 Proof of Theorem 1

sBM has superior equilibria to DA when there is competition for schools. To simplify exposition, we introduce a new school o with unlimited capacity ($q_o = \infty$). For each school $a \in A$ we will add a set of students that provide competition for each of the q_a seats at a. Specifically, for $i = 1, 2, \ldots, q_a$, let c_i^a be a new student whose favorite school is a and whose second favorite school is o: $P_{c_i^a} : a, o$. We will refer to each c_i^a as the competition for a. Because c_i^a is guaranteed her second choice, her weakly dominant strategy is to report her preferences truthfully under sBM.

Let $\mu = DA(\Gamma)$ and let ν be any Pareto efficient matching which Pareto dominates μ. Consider any student i such that $\mu(i) = \nu(i)$. Let $b = \mu(i)$. First, we add students to ensure i faces competition for school b. Next, we make sure i is not guaranteed school b. Specifically, add students p_j^b where $1 \leq j \leq q_b$ such that each p_j^b's favorite school is o (note that they are guaranteed o). Next, define b's priorities over this larger set of students as follows. Each $p_j^b \succ_b' i \succ_b' c_k^b$ for all j and k. Otherwise, keep the priorities at b the same as in $\succ_b$. Adding the p_j^b's ensures i is not guaranteed school b, and we add the competition students (c_k^b's) so that i can only be assigned b if she ranks it first.

Now consider any student j such that $\nu(j) P_j \mu(j)$. We proceed with a similar construction. Let $a = \mu(j)$. Any student k such that $k \succ_a j$ is either assigned under μ to a or a school she strictly prefers. If she is assigned to a school she prefers to a, then lower her priority under $\succ_a'$ to below j. Note that when we run DA on the new priorities, this will not change any assignment as k never applies to a under either priority and therefore her priority at a is irrelevant to the assignment. But under $\succ_a'$, j now has one of the q_a highest priorities at a. Let $\nu(j) = b$. Next, as before add q_b students p_k^b that each rank o first but have higher priority at b than j. The key point is that j is not guaranteed $\nu(j)$, j will not be assigned to $\nu(j)$ if she does not rank it first, but j is guaranteed $\mu(j)$.

We claim that the following set of strategies constitutes a Nash equilibrium in this larger economy under sBM. Each new student submits truthful preferences (this is a dominant strategy by construction). Each student i such that $\mu(i) = \nu(i)$ ranks $\mu(i)$ first (we will refer to these as Type 1 students). Each student i such that $\nu(i) P_i \mu(i)$ ranks $\nu(i)$ first and $\mu(i)$ second (we will refer to these as Type 2 students). Consider a Type 1 student i. μ is a stable assignment, so if i prefers a school a to $\mu(i)$, then each student μ assigns to a has higher priority at a than does i. Suppose for contradiction that i could receive a better assignment by applying to a school $bP_i\mu(i)$ in the first round. By applying to b, i must have caused a student j to be rejected (or else ν would not be Pareto efficient). The Type 1 students that apply to b all have higher priority at b than i does, so j must be a Type 2 student. j now applies to $\mu(j)$. Since she is guaranteed $\mu(j)$, this causes a student k to be rejected by $\mu(j)$ in the second round.

We claim k must be a Type 2 student. If k were a Type 1 student, then there is some Type 2 student l that is not rejected by $\mu(j)$. But l prefers $\mu(j)$ to $\mu(l)$, and since $\mu(j)$ rejected k instead of l, l must have higher priority at $\mu(j)$ than k does. But k is a Type 1 student, so $\mu(k) = \mu(j)$. But this means that l has justified envy of k at $\mu(j)$, which is a contradiction.

To summarize, i, a Type 1 student, applied to b in the first round. This caused j, a Type 2 student, to be rejected. When j applies to $\mu(j)$, this causes a Type 2 student k to be rejected by $\mu(j)$. By repeating the above logic, we see that each new application causes a Type 2 student to be rejected. This process must eventually stop, but it only stops when a Type 2 student applies to b and a Type 1 student is rejected by b. Since i has the lowest priority at b of all the Type 1 students, it must be that i is eventually rejected by b. Note that by applying to b instead of $\mu(i)$, i will not be accepted to $\mu(i)$ if she applies there in a later round. Since i will not be assigned to a school she prefers to $\mu(i)$, applying to any school she prefers to $\mu(i)$ can only harm her eventual assignment.

A similar argument applies to Type 2 students. If a Type 2 student i applies to a school b that she prefers to $\nu(i)$, she is either rejected immediately or eventually by that school. However, by applying to a school other than $\nu(i)$ in the first round, she will not be accepted to $\nu(i)$ if she applies there in a later round and must end up being assigned to $\mu(i)$, a school she finds strictly inferior.

Finally, note that none of the new students or changes to school priorities change the way students are assigned to the objects in A under DA.

A.2 Proof of Theorem 2

We first show that for any problem P, if BM and sBM select different (tentative) outcome in any step, then there exists at least one student who can manipulate BM. Let μ and ν be the matchings selected by BM and sBM, respectively. Then, by the definition of BM and sBM, in some step $k > 1$ of sBM a tentatively accepted student in step $k-1$ by some school s is rejected because one of the students in G_s applies to s in step k. Without loss of generality, suppose k is the first step in which a tentatively accepted student in an earlier step is rejected. Denote the number of students in G_s applying to s in step k by α. Since some tentatively accepted students are rejected in this step the number of student tentatively and permanently accepted by school s in step $k-1$ is larger than $q_s - \alpha$. Since all the steps up to k are the same in both BM and sBM, at least one student in G_s who applies to s in step k is rejected by s under BM and assigned to a worse school than s under BM. Denote this student by i. Since i has one of the top q_s priority for s, she will be assigned to s if she ranks s at the top of her preference list under BM.

Now suppose that BM and sBM select the same outcome for problem P in all steps and there exists a student i who can manipulate sBM. Since in all steps BM and sBM select the same tentative outcome then they select the same final outcome. Let μ be the outcome selected by both BM and sBM under truth-telling. Let s be the school that i can be assigned to when she misreports under sBM. That is, $sP_i\mu(i)$. We can see that $i \notin G_s$. Moreover, since all the steps of BM and sBM are the same, none of the tentatively assigned students is rejected when a guaranteed student applies under sBM. Hence, when i applies to s either all the seats at s were assigned in an earlier step or the number of applicants with higher priority in that step is more than the number of remaining seats. The only way that i can get s is ranking s in a higher place in her submitted list. Since she can be assigned in an earlier step, the number of students with higher priority applying to s in the first step is less than the number of available seats and i can get s if she ranks it at the top of her preference list under BM.

Now we show that there exists at least one problem in which BM is manipulable but sBM is not. Let $S = \{a, b, c\}$, $I = \{i, j, k\}$, and $q = (1, 1, 1)$. The preferences and priorities are:

i	j	k	a	b	c
a	a	b	i	j	k
	b	a	j	k	
	c	c			

sBM assigns i to a, j to b, and k to c under truth-telling. The only student who is not assigned to her top choice, k, cannot change the outcome of sBM by reporting some other preference list. BM assigns i to a, k to b, and j to c under truth-telling. However, j will be assigned to b if she reports it as her top choice.

A.3 Proof of Proposition 2

Consider a problem P in which BM selects a fair matching. Since, the outcome of the BM is fair in any round k, there does not exist a school receiving an application from a student with higher priority than the ones it permanently accepted in a previous round. By the definition of sBM, in each round the same students will apply to the same schools in problem P and sBM selects the same outcome as BM. Hence, sBM selects a fair matching whenever BM does.

Now consider Example 3; under the reported preferences, sBM selects a fair matching but under BM's outcome student i's priority at school b is violated.

A.4 Proof of Proposition 3

Consider a problem P in which BM and sBM select different outcomes μ and ν, respectively. By the definition of sBM, there exists at least a student-school pair (i, s) such that $i \in G_s$ and $\nu(i) P_i \mu(i)$.

B Experimental Instructions

B.1 Links to Videos Accompanying the Instructions

Secure Boston Mechanism videos

1. http://youtu.be/2KBXJ0W38q4
2. http://youtu.be/5ylYh6sTgCg
3. http://youtu.be/nh63zAxxRa4

Boston Mechanism videos

1. http://youtu.be/4mexuRDK8mc

2. http://youtu.be/CpYm4tRfRiA

3. http://youtu.be/zw8c1W0mIYE

B.2 Instructions for the Secure Boston Mechanism

Welcome! Today you will participate in an experiment where you can earn a considerable amount of money that will be paid to you in cash as you leave today. The following instructions tell you everything you need to know to earn as much as possible, so please read them carefully. If you have any questions, please raise your hand and a monitor will come and answer them.

You will play the role of a student choosing a school. You will rank schools in order to try to get a seat at the best school you are able. You will earn points based on how much you like the school at which you get a seat, where the number of points at a school is called your payoff at that school. These payoffs tell you how much you like each school.

There are two different settings in today's experiment. For some periods, you will be a student choosing among Four Schools; in other periods, you will be a student choosing among Eight Schools. You will always know the number of schools as you decide how to rank schools. Further, you will always have a copy of these instructions to refer to and you are encouraged to look over the instructions again during the experiment.

You will participate in 32 periods in today's experiment. After the last period, the computer will randomly choose two periods to count for your payment. One of the randomly chosen periods will be from a Four Schools period and the other will be from an Eight Schools period. Any one of the periods could be one of the periods that count! Treat each period as if it could be one that determines your payment.

All points that you earn in today's experiment will be converted to American dollars and you will be paid in cash at the end of the experiment. Every two points equal $1 American dollar. Your goal is to earn as many points as possible in order to earn as much cash as possible!

The other participants in your group will be played by the computer, so no one else in the room is in your group and instead everyone else in the room is in their own group.

Four Schools

There are four schools: A, B, C, and D. Each school has 3 available seats. There are a total of 12 students (including you) trying to gain admission to one of the schools. Your payoff for each school changes from period to period but you will always be told your payoff for each school as you decide how to rank the schools. Which school has the highest, second highest, etc. payoff for you changes from period to period but the payoffs are always as follows:

- Highest payoff = 20 points
- Second highest payoff = 10, 11, 12, 13, 14, or 15 points (each equally likely)
- Third highest payoff = 4, 5, 6, 7, 8 or 9 points (each equally likely)

- Lowest payoff = 0 points

Being assigned to a school that gives you a higher payoff earns you more points and therefore more cash in your pocket!

Eight Schools

There are eight schools: A, B, C, D, E, F, G, and H. Each school has 30 available seats. There are a total of 240 students (including you) trying to gain admission to one of the schools. Your payoff for each school changes from period to period but you will always be told your payoff for each school as you decide how to rank the schools. Which school has the highest, second highest, etc. payoff for you changes from period to period but the payoffs are always as follows:

- Highest payoff = 20 points
- Second highest payoff = 14, 15, 16, 17, or 18 points (each equally likely)
- Third highest payoff = 9, 10, 11, 12, or 13 points (each equally likely)
- Fourth highest payoff = 7 or 8 points (each equally likely)
- Fifth highest payoff = 5 or 6 points (each equally likely)
- Sixth highest payoff = 3 or 4 points (each equally likely)
- Seventh highest payoff = 1 or 2 points (each equally likely)
- Lowest payoff = 0 points

Being assigned to a school that gives you a higher payoff earns you more points and therefore more cash in your pocket!

As mentioned earlier, the other participants in your group will be played by the computer, so no one else in the room is in your group and instead everyone else in the room is in their own group.

The lists that are submitted by the computer participants in your group are randomly determined by the computer. You will be shown the number of computer participants that rank each school first on the list they submitted. The lists of the computer participants change from period to period but you will always be told the exact number of computer participants that rank each school first in that period. For each computer participant, the schools ranked second, third, etc. are also randomly determined by the computer and are representative of what you are shown with the first choices.

Some schools are more popular than other schools. You will be able to tell which schools are popular because you are shown the number of computer participants that rank each school first. Which schools are popular changes from period to period but you will always be shown the number of first choices.

At each school, you have a priority that affects the order in which your application will be considered at that school. Your priority at each school depends on three things:

- Whether you live within the school district of that school,
- The place you rank that school in your list, and
- Your Lottery Number, which is used for tie-breaking.

Each student (you and the computer participants) lives in the district of one of the schools, which, as you see above, affects the priority order. For the entire experiment, you will be a student who lives within the school district of school A. Each computer participant is a district student at some school and each school has the same number of district students as its capacity (3 in the 4 schools setting, 30 in the 8 schools setting).

Specifically, the priority order of the students for each school is determined as follows:

- First Priority Level: Participants who live within the school district. Since the number of participants living within the each school district is equal to the school capacity in that district, each **First Priority** participant is guaranteed to be assigned to a school at least good as his/her district school.
- Second Priority Level: Participants who rank the school as their **first choice** and who do not live within the school district.
- Third Priority Level: Participants who rank the school as their **second choice** and who do not live within the school district.
- $\cdots$

Your Lottery Number comes from a fair lottery that is used to break ties between participants at the same priority level. This means each participant at a given priority level has an equal chance of being the first in line, second in line, , as well as last in line.

In this fair lottery, each student is assigned a number between 0 and 1, with each number being equally likely. A student whose number is closer to 1 has a higher chance of being in the front of the line among those at the same priority level.

In half of the periods in today's experiment, you will be told your Lottery Number as you decide how to rank the schools. In the other half of the periods, you will not be told your Lottery number. Before the first period, you will be told whether you will see your Lottery Number in the first half or in the second half.

Your Lottery Number changes each period, whether or not you are told your number. It is always a number between 0 (lowest priority) and 1 (highest priority), with each number being equally likely.

When you submit your rankings of schools, the system takes your rankings with those of the computer participants in your group in order to determine which students get which seats. This process works as follows:

- An application to the first choice school is sent for each participant (you and the computer participants).
- Throughout the process, a school cannot hold more applications than its capacity. If a school receives more applications than its capacity, then it rejects the students with the lowest priorities. The remaining students are retained.
- Whenever a student is rejected at a school, his/her application is sent to the next highest (best) school on his/her list.
- Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the school's capacity are rejected, while remaining applications are retained.
- The process ends when all schools are at their capacity (all seats are taken).
- Each student is assigned a seat at the school that holds his/her application at the end of the process.

Each period, you will have up to three minutes to submit your rankings of schools.

Please raise your hand and a monitor will start the first video for you.

An Example: We will go through an example. This example is not meant to fit the number of students and schools that you will see. Instead, it is meant to illustrate how the process works. You will be asked to work out the assignments in this example for Review Question 1.

Feel free to refer to the experimental instructions before you answer any question. Each correct answer is worth half a point, which will be added to your total earnings. You can earn up to 6.5 points for the review questions.

Students and Schools: In this example, there are four students, 1-4, and four schools, A, B, C, and D.

Student ID Number: 1; 2; 3; 4 Schools: A, B, C, D

Slots and Residents: There is one slot at each school. Residents of districts are indicated in the table below.

School	Slot	District Residents
A		1
B		2
C		3
D		4

Lottery: The lottery assigns the following number to the students:

	Lottery Numbers
Student 1	.95
Student 2	.67
Student 3	.30
Student 4	.05

Submitted School Rankings: The students submit the following school rankings:

	1st Choice	2nd Choice	3rd Choice	Last Choice
Student 1	D	A	C	B
Student 2	D	A	B	C
Student 3	A	B	C	D
Student 4	A	D	B	C

Priority: School priorities first depend on the district resident, second on the place ranked on preference list, and next on the lottery order:

Priority order at A: $\overbrace{1}^{\text{Resident}}\ \overbrace{-3-4-2}^{\text{Non-Residents}}$

Priority order at B: $2\ -3-4-1$

Priority order at C: $3\ -1-2-4$

Priority order at D: $4\ -1-2-3$

The process consists of the following steps: Please use this sheet to work out the allocation and enter it into the computer for Review Question #1.

Step 1: Each student applies to his/her first choice. If a school receives more applications than its capacity, then it rejects the students with the lowest priorities. The remaining students are retained.

Applicants	School	Accept	Hold	Reject
→	A	→ N/A		
→	B	→ N/A		
→	C	→ N/A		
→	D	→ N/A		

Step 2: Each student rejected in Step 1 applies to his/her next highest (best) school. Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the school's capacity are rejected, while remaining applications are retained.

Held	Applicants	School	Accept	Hold	Reject
	→	A	→ N/A		
	→	B	→ N/A		
	→	C	→ N/A		
	→	D	→ N/A		

Step 3: Each student rejected in Step 2 applies to his/her next highest (best) school. Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the school's capacity are rejected, while remaining applications are retained.

Held	Applicants	School	Accept	Hold	Reject
	→	A	→ N/A		
	→	B	→ N/A		
	→	C	→ N/A		
	→	D	→ N/A		

Step 4: Each student rejected in Step 3 applies to his/her next highest (best) school. Whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the lowest priority ones in excess of the school's capacity are rejected, while remaining applications are retained.

Held	Applicants	School	Accept	Hold	Reject
	→	A	→ N/A		
	→	B	→ N/A		
	→	C	→ N/A		
	→	D	→ N/A		

The process ends at Step 5.

Held	Applicants	School	Accept	Hold	Reject
	→	A	→	N/A	N/A
	→	B	→	N/A	N/A
	→	C	→	N/A	N/A
	→	D	→	N/A	N/A

Please enter your answer to the computer for Review Question 1. Feel free to look through the instructions and the example as you complete this and the remaining Review Questions.
Correct Answer for sBM: Student 1 is allocated to School A, 2 to B, 3 to C, and 4 to D.
Correct Answer for BM: Student 1 is allocated to School D, 2 to B, 3 to A, and 4 to C.

[The second video automatically starts here.]

Afterwards, you will be asked to answer another 9 review questions.

Review Questions 2 - 10

2. How many computer participants are in your group in the setting with 4 Schools? *Correct Answer: 11*
3. True or false: None of the other people in this room are in your group. *Correct Answer: True*
4. True or false: A student who lives in a school's district and ranks the school first has higher priority than a student who does not live in the district and ranks the school first. *Correct Answer: True*
5. Suppose that Student 1's district school is school C and Student 2's district school is school D. If Student 1 ranks school C second on her list and Student 2 ranks school C first on his list, which student has a higher priority at school C? *Correct Answer for sBM: Student 1, Correct Answer for BM: Student 2*
6. True or false: Your Lottery Number is the same for the entire 32 periods. *Correct Answer: False*
7. True or false: Whether or not you are shown your Lottery Number, it is always a number between 0 (lowest priority) and 1 (highest priority), with each number being equally likely. *Correct Answer: True*
8. True or false: You will be shown how many of the computer participants rank each school first in each period. *Correct Answer: True*
9. True or false: You can be assigned to a school that gives you fewer points than your district school. *Correct Answer for sBM: False, Correct Answer for BM: True*
10. True or false: Your district school changes from period to period. *Correct Answer: False*

As a reminder, each correct answer is worth half a point, which will be added to your total earnings. You can earn up to 6.5 points for the review questions.

Of the 13 review questions, you need to answer at least 10 correctly in order to continue to the experiment. If you answer fewer than 10 questions correctly, you will be asked to retake the quiz until you correctly answer at least 10 questions. However, you will only be paid for the number of questions correct on the first quiz attempt. You may ask questions at any point and may refer to the instructions and the example throughout the quiz and the experiment itself.

If you have any questions, please raise your hand and a monitor with come and answer them.

[The third video automatically starts here.]

B.3 Instructions for the Boston Mechanism

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Specifically, the priority order of the students for each school is determined as follows:

- First Priority Level: Participants who rank the school as their first choice and who live within the school district.
- Second Priority Level: Participants who rank the school as their first choice and who do not live within the school district.
- Third Priority Level: Participants who rank the school as their second choice and who live within the school district.
- Fourth Priority Level: Participants who rank the school as their second choice and who do not live within the school district.
- $\cdots$

$\ldots$

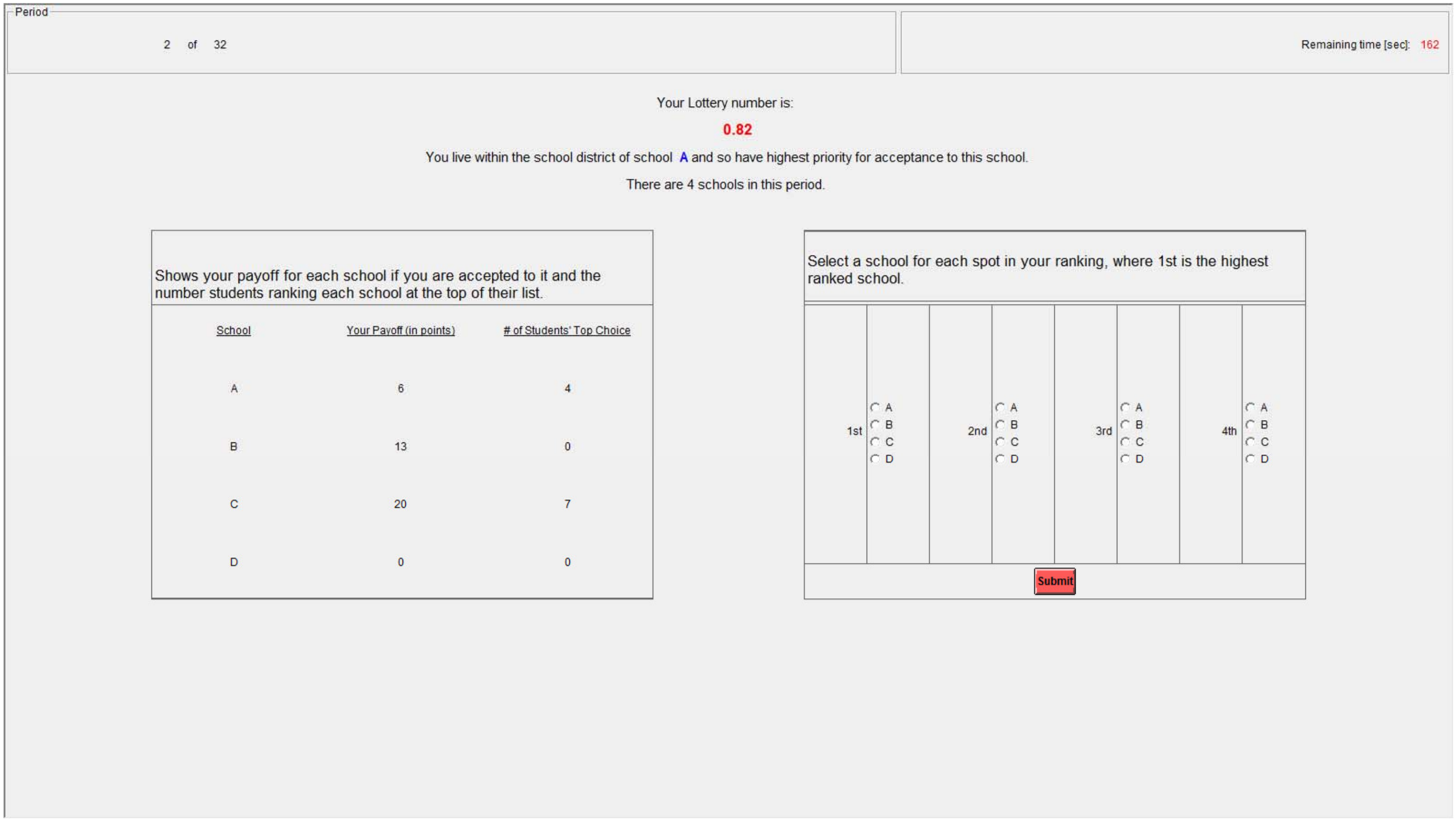

Figure 1: Screenshot of Experimental Interface, Ranking Screen

School Options Information (click on school name for more information)

MAGNET SCHOOLS	Program Description	Calendar	Sibling Priority	Transportation	Current 1st Choice Applicants
A. B. Combs Magnet Elementary (XC)	Combs Leadership	Traditional		Parent	13
Brentwood Magnet Elementary (EN)	Engineering Magnet	Traditional		Express	4
Brooks Museums Magnet Elementary (MU)	Museums Magnet	Traditional		Parent	12
Bugg Magnet Elementary (CA)	Creative Arts and Science Program	Traditional		Express	13
Conn Magnet Elementary (GC)	Active Learning and Technology	Traditional		Express	3
Fuller Magnet Elementary (GT/AG)	Gifted and Talented/AG Basics Program	Traditional		Neighborhood	14
Green Elementary (LW)	Leadership and World Languages	Traditional		Parent	3
Hodge Road Elementary (DS)	Spanish Dual Language Immersion	Year Round		Parent	1
J. Y. Joyner Magnet Elementary (LA)	Center for Spanish Language/International Baccalaureate Primary Years Programme	Traditional		Parent	4
Jeffreys Grove Elementary (SI)	Spanish Immersion	Traditional		Parent	1
Kingswood Elementary (MT)	Montessori Program grades PK-03; STEM grades 04-05	Traditional		Parent	3
Partnership Elementary (PE)	School of Choice	Modified		Express	1
Stough Elementary (CI)	Chinese Immersion	Traditional		Parent	3
Washington Magnet Elementary (GT)	Gifted and Talented Program	Traditional		Neighborhood	16
Wiley Magnet Elementary (IN)	International Studies	Traditional		Parent	5

Figure 2: Screenshot of an Application Website from the Field

Table 1: Treatment Randomization, Summary Statistics

	sBM Mechanism	BM Mechanism
Session	2.483	2.621
	(1.056)	(1.208)
Passed Quiz	0.655	0.655
	(0.484)	(0.484)
Demand at Favorite	0.513	0.512
	(0.029)	(0.031)
Favorite Preference Intensity	5.739	5.766
	(0.330)	(0.267)
District Second Favorite	0.584	0.611
	(0.078)	(0.079)
Risk Averse Subject	0.483	0.448
	(0.509)	(0.506)
Ambiguity Averse Subject	0.207	0.379
	(0.412)	(0.494)
Past Experiments	0.138	0.103
	(0.351)	(0.310)
Male Subject	0.621	0.414
	(0.494)	(0.501)
Age	19.793	19.552
	(1.760)	(1.785)
Economics Major	0.034	0.069
	(0.186)	(0.258)
American Subject	1.000	0.966
	(0.000)	(0.186)
White Subject	0.724	0.724
	(0.455)	(0.455)
Need-Based Aid	0.379	0.345
	(0.494)	(0.484)
SAT Score	1240.690	1223.793
	(147.477)	(133.642)
High School GPA	4.339	4.311
	(0.398)	(0.349)

Notes: Variable means are shown, with standard deviations in parentheses. None of the variables are statistically significantly different across the sBM and BM mechanisms.

Table 2: Truth-Telling, Summary Statistics

	(1) $true_1$	(2) $true_d$	(3) $true_a$	(4) $true_{cs}$
sBM Mechanism	0.380	0.284	0.176	0.284
	(0.014)	(0.013)	(0.011)	(0.013)
BM Mechanism	0.274	0.171	0.127	0.127
	(0.012)	(0.010)	(0.009)	(0.009)
Difference	0.106	0.112	0.048	0.156
	(0.018)***	(0.016)***	(0.014)***	(0.016)***
N	2592	2592	2592	2592

Notes: Four measures of truth-telling are shown: $true_1$, submitting one's favorite school first; $true_d$, submitting the preference ranking truthfully up to the student's district school; $true_a$, submitting the entire preference ranking truthfully; and, following Chen and Sönmez (2006), $true_{cs}$, submitting the entire preference ranking truthfully for BM and submitting the preference ranking truthfully up to the student's district school for sBM. For this and subsequent tables, standard errors are in parentheses; $*$, $**$, and $***$ denote significance at the 10%, 5%, and 1% level, respectively. For this and subsequent means tables, the significance markers are from nonparametric tests, as explained in the text.

Table 3: Truth-Telling, Regression Results

	All 32 Periods		Final 16 Periods	
	(1)	(2)	(3)	(4)
sBM Mechanism	0.117	0.125	0.101	0.130
	(0.020)***	(0.022)***	(0.022)***	(0.024)***
Four Schools	0.116	0.114	0.107	0.089
	(0.025)***	(0.026)***	(0.037)***	(0.043)**
Tie Breaker Shown	0.004	0.013	-0.001	-0.028
	(0.016)	(0.017)	(0.022)	(0.029)
Period	0.001	0.000	0.001	-0.003
	(0.001)	(0.001)	(0.002)	(0.003)
Second Session	0.004	0.018	0.089	0.099
	(0.024)	(0.029)	(0.031)***	(0.030)***
Third Session	0.041	0.046	0.081	0.064
	(0.032)	(0.033)	(0.032)**	(0.028)**
Fourth Session	0.045	0.107	0.089	0.143
	(0.026)*	(0.035)***	(0.032)***	(0.042)***
Passed Quiz	0.007	0.022	0.032	0.063
	(0.022)	(0.019)	(0.022)	(0.019)***
Demand at Favorite	-0.015	0.021	0.061	0.123
	(0.054)	(0.063)	(0.066)	(0.075)*
Favorite Preference Intensity	0.012	0.009	0.016	0.011
	(0.005)**	(0.006)*	(0.007)**	(0.009)
District Second Favorite	0.094	0.092	0.082	0.099
	(0.016)***	(0.018)***	(0.022)***	(0.026)***
Subject Covariates	No	Yes	No	Yes
Observations	2592	1856	1296	928
Log Likelihood	-1,294.499	-904.724	-645.685	-445.027

Notes: The dependent variable is $true_d$, submitting the preference ranking truthfully up to the student's district school. The regression specification is a random-effects logit model with subject-level random effects. Marginal effects at the mean are shown along with heteroskedasticity-robust standard errors. Demographic and other subject-level covariates are included in the regressions but their results are shown in Table 12.

Table 4: Truth-Telling and Position of the District School

	(1) 2nd Favorite	(2) 3rd Favorite	(3) 4th Favorite
sBM Mechanism	0.342	0.244	0.029
	(0.017)	(0.021)	(0.016)
BM Mechanism	0.173	0.211	0.000
	(0.013)	(0.020)	(0.000)
Difference	0.169	0.034	0.029
	(0.022)***	(0.029)	(0.016)*
N	1537	848	207

Notes: Truth-telling is measured by $true_d$, submitting the preference ranking truthfully up to the student's district school. The position of the district schools shows where the subject's district school falls in her true preference ranking.

Table 5: District School Bias, Summary Statistics

	(1) dsb_1	(2) dsb
sBM Mechanism	0.048	0.204
	(0.006)	(0.011)
BM Mechanism	0.371	0.512
	(0.013)	(0.014)
Difference	-0.323	-0.308
	(0.015)***	(0.018)***
N	2592	2592

Notes: Two measures of district school bias are shown: dsb_1, submitting one's district school first and dsb, submitting one's district school higher than if truthful.

Table 6: District School Bias, Regression Results

	All 32 Periods		Final 16 Periods	
	(1)	(2)	(3)	(4)
sBM Mechanism	-0.317	-0.328	-0.311	-0.313
	(0.024)***	(0.027)***	(0.028)***	(0.027)***
Four Schools	-0.045	-0.062	-0.046	-0.047
	(0.031)	(0.030)**	(0.036)	(0.040)
Tie Breaker Shown	0.047	0.049	0.071	0.076
	(0.018)***	(0.022)**	(0.029)**	(0.032)**
Period	-0.002	-0.002	-0.002	-0.001
	(0.001)*	(0.001)	(0.003)	(0.003)
Second Session	-0.022	-0.037	-0.049	-0.065
	(0.032)	(0.040)	(0.039)	(0.049)
Third Session	-0.017	-0.029	-0.062	-0.073
	(0.036)	(0.039)	(0.039)	(0.042)*
Fourth Session	-0.016	-0.082	-0.009	-0.080
	(0.031)	(0.032)**	(0.035)	(0.041)**
Passed Quiz	-0.024	-0.029	-0.052	-0.042
	(0.027)	(0.024)	(0.029)*	(0.027)
Demand at Favorite	0.174	0.171	0.143	0.186
	(0.058)***	(0.069)**	(0.080)*	(0.095)*
Favorite Preference Intensity	-0.016	-0.014	-0.019	-0.020
	(0.006)***	(0.006)**	(0.008)**	(0.010)**
District Second Favorite	-0.069	-0.065	-0.080	-0.100
	(0.037)*	(0.046)	(0.042)*	(0.051)*
Subject Covariates	No	Yes	No	Yes
Observations	2592	1856	1296	928
Log Likelihood	-1,461.375	-1,035.406	-701.347	-492.193

Notes: The dependent variable is *dsb*, submitting one's district school higher than if truthful. The regression specification is a random-effects logit model with subject-level random effects. Demographic and other subject-level covariates are included in the regressions but their results are shown in Table 12.

Table 7: First Choice Accommodation: Reported Preferences

	(1) 1st	(2) 2nd	(3) 3rd	(4) 4th	(5) 5th
sBM Mechanism	0.369	0.861	0.997	1.000	1.000
	(0.013)	(0.010)	(0.002)	(0.000)	(0.000)
BM Mechanism	0.727	0.889	0.953	0.992	0.996
	(0.012)	(0.009)	(0.006)	(0.002)	(0.002)
Difference	-0.358	-0.029	0.044	0.008	0.004
	(0.018)***	(0.013)**	(0.006)***	(0.002)***	(0.002)**
N	2592	2592	2592	2592	2592

Notes: Shown are the cumulative probabilities of being assigned to a school at or above a given position in the subject's *reported* preference list.

Table 8: First Choice Accommodation: True Preferences

	(1) 1st	(2) 2nd	(3) 3rd	(4) 4th	(5) 5th	(6) 6th	(7) 7th
sBM Mechanism	0.122	0.467	0.792	0.906	0.927	0.955	0.979
	(0.009)	(0.014)	(0.011)	(0.008)	(0.007)	(0.006)	(0.004)
BM Mechanism	0.127	0.457	0.747	0.892	0.916	0.937	0.970
	(0.009)	(0.014)	(0.012)	(0.009)	(0.008)	(0.007)	(0.005)
Difference	-0.005	0.010	0.045	0.014	0.011	0.019	0.009
	(0.013)	(0.020)	(0.017)***	(0.012)	(0.011)	(0.009)**	(0.006)
N	2592	2592	2592	2592	2592	2592	2592

Notes: Shown are the cumulative probabilities of being assigned to a school at or above a given position in the subject's *true* preference list.

Table 9: Assignment Relative to the District School: True Preferences

	(1) More Preferred	(2) District School	(3) Less Preferred
sBM Mechanism	0.421	0.561	0.018
	(0.014)	(0.014)	(0.004)
BM Mechanism	0.423	0.487	0.090
	(0.014)	(0.014)	(0.008)
Difference	-0.002	0.074	-0.072
	(0.019)	(0.020)***	(0.009)***
N	2592	2592	2592

Notes: Shown are the probabilities of being assigned to a school that is more preferred, equal to, or less preferred than the subject's district school in her *true* preference list.

Table 10: Instances of Justified Envy (Fairness), Regression Results

	All 32 Periods		Final 16 Periods	
	(1)	(2)	(3)	(4)
sBM Mechanism	-0.442	-0.588	-0.400	-0.471
	(0.144)***	(0.122)***	(0.150)***	(0.183)***
Four Schools	-1.150	-0.895	-0.696	-0.579
	(0.198)***	(0.197)***	(0.182)***	(0.231)**
Tie Breaker Shown	0.031	-0.105	0.067	0.116
	(0.129)	(0.133)	(0.145)	(0.208)
Period	-0.021	-0.016	-0.012	-0.017
	(0.008)**	(0.008)**	(0.017)	(0.021)
Second Session	-0.176	-0.318	-0.036	0.069
	(0.216)	(0.238)	(0.280)	(0.401)
Third Session	-0.296	-0.527	-0.353	-0.510
	(0.164)*	(0.218)**	(0.202)*	(0.291)*
Fourth Session	-0.087	-0.659	-0.069	-0.339
	(0.224)	(0.247)***	(0.231)	(0.342)
Passed Quiz	0.013	-0.054	-0.095	-0.005
	(0.157)	(0.170)	(0.182)	(0.291)
Demand at Favorite	-0.457	-0.801	-0.241	-0.310
	(0.250)*	(0.273)***	(0.306)	(0.437)
Favorite Preference Intensity	0.002	-0.047	-0.047	-0.097
	(0.042)	(0.045)	(0.049)	(0.059)*
District Second Favorite	-0.633	-0.541	-0.489	-0.443
	(0.154)***	(0.149)***	(0.140)***	(0.176)**
Subject Covariates	No	Yes	No	Yes
Observations	2592	1856	1296	928
R-Squared	0.045	0.059	0.044	0.052

Notes: The dependent variable is the number of instances of justified envy, that is, the number of robotic subjects who are assigned to a school that the human subject prefers, where the human subject has a higher priority. Higher levels of fairness are associated with lower levels of justified envy. The regression specification is a random-effects linear model with subject-level random effects. Demographic and other subject-level covariates are included in the regressions but their results are shown in Table 12.

Table 11: Efficiency, Regression Results

	All 32 Periods		Final 16 Periods	
	(1)	(2)	(3)	(4)
sBM Mechanism	0.217	0.456	0.151	0.296
	(0.144)	(0.151)***	(0.183)	(0.190)
Four Schools	0.751	0.775	0.732	0.796
	(0.192)***	(0.232)***	(0.271)***	(0.339)**
Tie Breaker Shown	0.070	0.032	0.292	0.407
	(0.121)	(0.127)	(0.181)	(0.214)*
Period	0.023	0.022	0.014	0.011
	(0.006)***	(0.007)***	(0.019)	(0.023)
Second Session	0.321	0.341	0.225	0.000
	(0.223)	(0.219)	(0.311)	(0.318)
Third Session	0.027	0.104	-0.031	-0.063
	(0.235)	(0.223)	(0.302)	(0.284)
Fourth Session	0.005	0.340	-0.027	0.132
	(0.209)	(0.206)*	(0.284)	(0.276)
Passed Quiz	0.220	0.131	0.328	0.234
	(0.142)	(0.149)	(0.186)*	(0.217)
Demand at Favorite	-4.974	-4.542	-5.484	-4.809
	(0.505)***	(0.533)***	(0.657)***	(0.719)***
Favorite Preference Intensity	-0.621	-0.646	-0.602	-0.664
	(0.049)***	(0.050)***	(0.068)***	(0.082)***
District Second Favorite	1.750	1.715	1.412	1.485
	(0.162)***	(0.179)***	(0.216)***	(0.262)***
Subject Covariates	No	Yes	No	Yes
Observations	2592	1856	1296	928
R-Squared	0.307	0.310	0.308	0.324

Notes: The dependent variable is the subject's payoff in points in each period, where two points are worth $1, which measures the level of efficiency of the allocation. The regression specification is a random-effects linear model with subject-level random effects. Demographic and other subject-level covariates are included in the regressions but their results are shown in Table 12.

Table 12: Demographic and Other Covariates, Regression Results

	(1) Truth-Telling	(2) District School Bias	(3) Fairness	(4) Efficiency
Risk Averse Subject	0.037	-0.060	-0.336	-0.194
	(0.021)*	(0.027)**	(0.133)**	(0.160)
Ambiguity Averse Subject	0.005	0.002	0.041	0.061
	(0.026)	(0.027)	(0.157)	(0.166)
Past Experiments	0.078	-0.049	-0.695	0.242
	(0.034)**	(0.058)	(0.269)***	(0.320)
Male Subject	0.015	0.043	0.292	-0.027
	(0.027)	(0.034)	(0.131)**	(0.163)
Age	-0.024	-0.003	-0.117	-0.054
	(0.010)**	(0.011)	(0.082)	(0.078)
Economics Major	0.248	-0.219	-0.592	0.248
	(0.063)***	(0.027)***	(0.232)**	(0.268)
American Subject	-0.100	0.085	-0.139	-1.085
	(0.070)	(0.063)	(0.424)	(0.427)**
White Subject	-0.111	0.093	0.565	-0.377
	(0.032)***	(0.030)***	(0.188)***	(0.180)**
Need-Based Aid	0.003	0.032	0.077	-0.175
	(0.022)	(0.032)	(0.145)	(0.168)
SAT Score	0.000	-0.000	0.000	0.001
	(0.000)	(0.000)	(0.001)	(0.001)
High School GPA	-0.032	0.001	-0.279	-0.172
	(0.035)	(0.056)	(0.251)	(0.251)
Observations	1856	1856	1856	1856

Notes: Shown are the demographic and other subject-level covariates from the earlier regressions, matching column (2) of Tables 3, 6, 10, and 11, respectively. Recall that higher levels of fairness are associated with lower levels of justified envy.

Table 13: Efficiency Comparison of sBM and DA

	Number Assigned			Cumulative Number Assigned		
	1^{st} choice	2^{nd} choice	3^{rd} choice	1^{st} choice	2^{nd} choice	3^{rd} choice
sBM Mechanism	8.016	3.293	0.691	8.016	11.309	12.000
	(0.033)	(0.027)	(0.014)	(0.033)	(0.014)	(0.000)
DA Mechanism	7.699	3.517	0.785	7.699	11.215	12.000
	(0.035)	(0.028)	(0.015)	(0.035)	(0.015)	(0.000)
Difference	0.317	-0.224	-0.094	0.317	0.094	0.000
	(0.012)***	(0.010)***	(0.005)***	(0.012)***	(0.005)***	(0.000)

Notes: Shown are the mean number of students assigned to each choice in their true preference ranking (or to a choice at this position or higher, cumulative), along with the standard errors in parentheses. The significance markers are from nonparametric tests, as explained in the text.