Suggested Solution to Problems 1

Problem 1

		Elizabeth: Stag	Elizabeth: Hare
Lou	Stag	$\underline{10}, \underline{10}$	$\underline{1}, 0$
Lou	Hare	$0, \underline{1}$	$\underline{1}, \underline{1}$

There are two pure-strategy Nash equilibria $(Stag, Stag)$ and $(Hare, Hare)$.

Problem 2

Players $i = Bob, Umut, Thayer$ choose $a_i \in$ {Attend (A), Not to attend (N)}.

Thayer choose A:

		Umut: A	Umut: N
Bob	A	$\underline{5}, \underline{5}, \underline{5}$	$\underline{5}, 0, \underline{5}$
Bob	N	$0, \underline{5}, \underline{5}$	$0, 0, -5$

Thayer choose N:

		Umut: A	Umut: N
Bob	A	$\underline{5}, \underline{5}, 0$	$-5, 0, 0$
Bob	N	$0, -5, 0$	$\underline{0}, \underline{0}, \underline{0}$

There are two pure-strategy Nash equilibria (A, A, A) and (N, N, N).

Problem 3

Players $i = Jerry, Duece$ choose $a_i \in [0, \infty)$. Note that a_i is an integer.

(i) Scenario 1: Suppose both $Jerry$ and $Duece$ gets the prize when $a_{Jerry} = a_{Duece}$.

- Player i's best response to Player j's actions:

$$BR_i(a_j) = \begin{cases} \{a_j,\ a_j+1,\ a_j+2, \ldots\} & , if \quad 0 \le a_j < 20 \\ \{0,\ 20,\ 21,\ \ldots\} & , if \quad a_j = 20 \\ \{0\} & , if \quad a_j > 20 \end{cases}$$

The set of Nash equilibria is the intersection of the best response functions, which are

- $(0,0)$, $(1,1)$ ... $(20,20)$, where $a_{Jerry} = a_{Duece}$ and $a_i \in [0,20]$
- $(20,0), (0,20), (0,21)$, $(21,0)$, $(0,22)$, $(22,0)$ ..., where $a_i \in [20,\infty)$ and $a_j = 0$

(ii) Scenario 2: Suppose neither of *Jerry* and *Duece* gets the prize when $a_{Jerry} = a_{Duece}$.

- Player i's best response to Player j's actions:

$$BR_i(a_j) = \begin{cases} \{a_j+1,\ a_j+2,\ a_j+3\ ...\} & ,if \quad 0 \le a_j < 20 \\ \{0,\ 21,\ 22\ ...\} & ,if \quad a_j = 20 \\ \{0\} & ,if \quad a_j > 20 \end{cases}$$

Nash equilibria are $(20,0), (0,20), (0,21)$, $(21,0)$, ..., where $a_i \in [20,\infty)$ and $a_j = 0$

Problem 4

Players $i = 1, 2$ choose $x_i \in [0,10]$. Note that x_i is an integer.

- Player i's best response to the other Player j's actions:

$$BR_i(x_j) = \begin{cases} \{10 - x_j,\ ...\ ,\ 10\} & ,if \quad 0 \le x_j < 6 \\ \{5,6\} & ,if \quad x_j = 6 \\ x_j - 1 & ,if \quad 6 < x_j \le 10 \end{cases}$$

Nash equilibria are $(5,6)$, $(6,5)$, $(5,5)$ and $(6,6)$.

Problem 5

(i) Suppose the person only cares about her own comfort.

		Player 2: Sit	Player 2: Stand
Player 1	Sit	$\underline{1}, \underline{1}$	$\underline{2}, 0$
	Stand	$0, \underline{2}$	$0, 0$

There is only one pure-strategy Nash equilibria (Sit, Sit).

(ii) Suppose the person is altruistic.

		Player 2: Sit	Player 2: Stand
Player 1	Sit	2, 2	0, <u>3</u>
Player 1	Stand	<u>3</u>, 0	<u>1</u>, <u>1</u>

There is only one pure-strategy Nash equilibria $(Stand, Stand)$.

Problem 6

Both players would try to maximize their payoff.

- Player 1's best response:

  $$x^*(y) = \underset{x}{\text{argmax}}(3 + 6y - \frac{3x}{2})x$$

  F.O.C: $3 + 6y - 3x = 0 \implies x^*(y) = 1 + 2y$

- Player 2's best response:

  $$y^*(x) = \underset{y}{\text{argmax}}(6 - x - \frac{y}{2})y$$

  F.O.C: $6 - x - y = 0 \implies y^*(x) = 6 - x$

Combine $x^*(y)$ and $y^*(x)$, we get $x^* = \dfrac{13}{3}$ and $y^* = \dfrac{5}{3}$.

The Nash equilibria is $\left(\dfrac{13}{3}, \dfrac{5}{3}\right)$