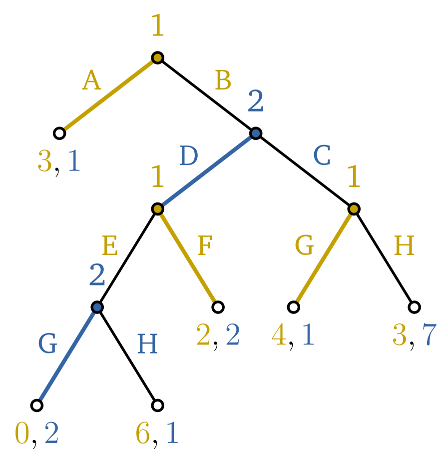
Suggested Solution to Problems 3

Problem 12

Player 1 has eight possible strategies: {AEG, AEH, AFG, AFH, BEG, BEH, BFG, BFH }.
Player 2 has four possible strategies: {DG, DH, CG, CH}



The backward induction equilibrium is (AFG, DG).

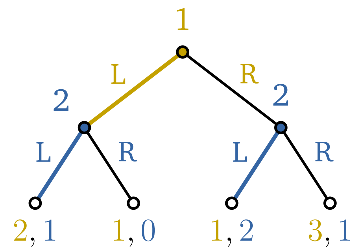
Problem 13

Player 1 has two possible strategies: {L ,R}.
Player 2 has four possible strategies: {LL, LR, RL, RR}

Player 1 \ Player 2	LL	LR	RL	RR
L	2, 1	2, 1	1, 0	1, 0
R	1, 2	3, 1	1, 2	3, 1

There are two Nash equilibria (L, LL) and (R, RL).

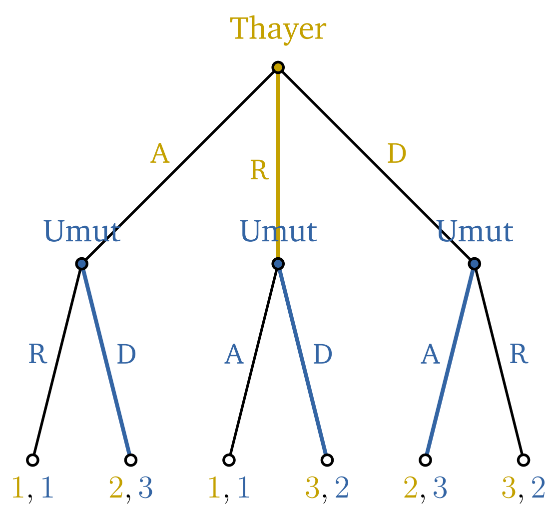
The subgame-perfect Nash equilibrium could be deduced by backward induction:



The subgame-perfect Nash equilibrium is (L, LL).

Problem 14

Thayer has three possible strategies: {Eliminate Action Movie (A), Eliminate Romantic Comedy (R), Eliminate Drama (D)}.
Umut has eight possible strategies {RAA, RDA, RAR, RDR, DAA, DDA, DAR, DDR }



They would watch action movie in equilibrium.

Problem 15

The subgame-perfect Nash equilibrium could be deduced by backward induction.

The last decision made in the game is made by Player 2.

Learning what player 1 choose, the dominant strategy for player 2 is:

$$S_2 = x_2^*(x_1) : \begin{cases} \{100 - x_1, \ldots, 100\} & , if \quad 0 \le x_1 < 51 \\ \{50,51\} & , if \quad x_1 = 51 \\ x_1 - 1 & , if \quad 51 < x_1 \le 100 \end{cases}$$

Using what player 2 has already determined will happen, player 1 thus needs to maximize:

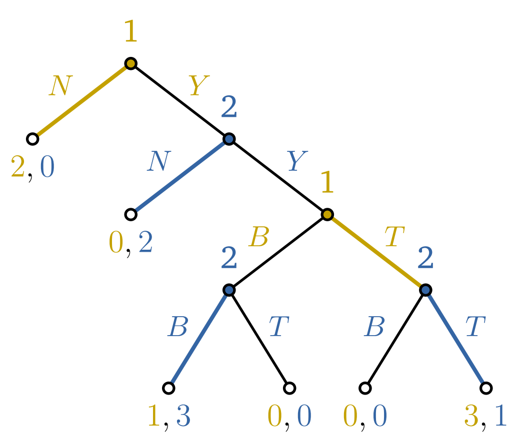
$$u_1(x_1, x_2^*(x_1)) = \begin{cases} x_1 & , if \quad 0 \le x_1 < 50 \\ 50 & , if \quad x_1 = 50, 51 \\ 101 - x_1 & , if \quad 51 < x_1 \le 100 \end{cases}$$

Along the SPNE path, we see that
- player 1 chooses 50 and player 2 chooses all numbers between 50 to 100
- player 1 chooses 51 and player 2 chooses 50 and 51.

Problem 16

The subgame-perfect Nash equilibrium could be deduced by backward induction.

We can present this sequential game as follows:



The SPNE is (NT, NBT).

Problem 17

The subgame-perfect Nash equilibrium could be deduced by backward induction.

The last decision made in the game is made by the firm.

Learning what the union chooses, the dominant strategy for the firm is to choose L that maximizes their profit conditional on w:

$\pi^*(w) = \underset{L}{\text{argmax}} \quad L(100 - L) - wL$

F.O.C: $100 - w - 2L = 0 \implies L^*(w) = \dfrac{100 - w}{2}$

$$S_{Firm} = L^*(w) : \begin{cases} \dfrac{100 - w}{2} & , if \quad 0 \leq w \leq 100 \\ 0 & , if \quad w > 100 \end{cases}$$

Using what the firm has already determined will happen, the union thus needs to maximize:

$u_{Union}(w, L^*(w)) = \underset{w}{\text{argmax}} \quad \dfrac{w(100 - w)}{2}$

F.O.C: $50 - w = 0 \implies w^* = 50$

Along the SPNE path, we see that the union chooses $w^* = 50$, and the firm chooses $L^* = 25$.