Suggested Solution to Problems 4

Problem 18

		Player 2 A	Player 2 B
Player 1	A	$6, 6$	$0, \underline{12}$
	B	$\underline{12}, 0$	$\underline{1}, \underline{1}$

Trigger Strategy for this game is:
Period 1: Play A
Period t: If in every earlier period we have each played A, then play A. If in any previous period either of us have ever played B, then B.

- $\pi^1_{cooperating} \geq \pi^1_{cheating} \implies \dfrac{6}{1-\delta} \geq 12 + \dfrac{\delta}{1-\delta}$

  Player 1 will cooperate so long as $\delta \geq \dfrac{6}{11}$

- $\pi^2_{cooperating} \geq \pi^2_{cheating} \implies \dfrac{6}{1-\delta} \geq 12 + \dfrac{\delta}{1-\delta}$

  Player 2 will cooperate so long as $\delta \geq \dfrac{6}{11}$

The trigger strategy will be a Nash equilibrium if $\delta \geq \dfrac{6}{11}$.

Problem 19

		Player 2 A	Player 2 B
Player 1	A	$2, 2$	$0, \underline{5}$
	B	$\underline{5}, 0$	$\underline{4}, \underline{4}$

Cooperation is advantageous when a NE is Pareto-inefficient. However, in the above game, there's no outcome better for both of the players than if each follow their own self-interests. They are not cooperating.

Problem 20

Player 2 (columns), Player 1 (rows)

	A	B
A	6, 6	0, 12
B	12, 0	1, 4

Trigger Strategy for this game is:
Period 1: Play A
Period t: If in every earlier period we have each played A, then play A. If in any previous period either of us have ever played B, then B.

- $\pi^1_{cooperating} \geq \pi^1_{cheating} \implies \dfrac{6}{1-\delta} \geq 12 + \dfrac{\delta}{1-\delta}$

  Player 1 will cooperate so long as $\delta \geq \dfrac{6}{11}$

- $\pi^2_{cooperating} \geq \pi^2_{cheating} \implies \dfrac{6}{1-\delta} \geq 12 + \dfrac{4\delta}{1-\delta}$

  Player 2 will cooperate so long as $\delta \geq \dfrac{3}{4}$

The trigger strategy will be a Nash equilibrium if $\delta \geq \dfrac{3}{4}$.

Problem 21

Player 2 (columns), Player 1 (rows)

	A	B
A	6, 6	0, 9
B	12, 0	1, 1

Trigger Strategy for this game is:
Period 1: Play A
Period t: If in every earlier period we have each played A, then play A. If in any previous

period either of us have ever played B, then B.

- $\pi^1_{cooperating} \geq \pi^1_{cheating} \implies \dfrac{6}{1-\delta} \geq 12 + \dfrac{\delta}{1-\delta}$

  Player 1 will cooperate so long as $\delta \geq \dfrac{6}{11}$

- $\pi^2_{cooperating} \geq \pi^2_{cheating} \implies \dfrac{6}{1-\delta} \geq 9 + \dfrac{\delta}{1-\delta}$

  Player 2 will cooperate so long as $\delta \geq \dfrac{3}{8}$

The trigger strategy will be a Nash equilibrium if $\delta \geq \dfrac{6}{11}$.

Problem 22

Firm $i = 1, 2$

In equilibrium, both firms would try to maximize their payoff.
Firm i's best response:
$q_i^*(q_j) = \underset{q_i}{\text{argmax}}(100 - q_i - q_j)q_i$

F.O.C: $100 - 2q_i - q_j = 0 \implies q_i^*(q_j) = \dfrac{100 - q_j}{2}$

Combine $q_1^*(q_2)$ and $q_2^*(q_1)$, we get $q_1^* = q_2^* = \dfrac{100}{3}$ and $\pi_1^* = \pi_2^* = \dfrac{10000}{9}$.
The Nash equilibria is $\left(\dfrac{100}{3}, \dfrac{100}{3}\right)$

Suppose the firms play this game an infinite number of times.
Firm $i = 1, 2$ choose to {(C)ooperate, (D)eviate}

If cooperate, they maximize their payoff by:
$Q^* = \underset{Q}{\text{argmax}}(100 - Q)Q$

F.O.C: $100 - 2Q = 0 \implies Q^* = 50$

$q_1^* = q_2^* = \frac{Q^*}{2} = \frac{50}{2} = 25$ and $\pi_1 = \pi_2 = 1,250$

Period 1: Firm 1 and Firm 2 play C and choose quantity 25
Period t: If in every earlier period both firm play C, then play C. If in any previous period either of the firms have ever play D, then play D.

If firm i choose to deviate, then firm i's and firm j's payoff would be:
$q_i^* = \underset{q_i}{\text{argmax}}(100 - 25 - q_i)q_i$

F.O.C: $75 - 2q_i = 0 \implies q_i^* = 37.5$
$\pi_1 = 1406.25$ and $\pi_2 = 937.5$

		Firm 2: C	Firm 2: D
Firm 1	C	1250, 1250	937.5, <u>1406.25</u>
	D	<u>1406.25</u>, 937.5	<u>1111.11</u>, <u>1111.11</u>

- $\pi^1_{cooperating} \geq \pi^1_{cheating} \implies \frac{1250}{1-\delta} \geq 1406.25 + \frac{10000\delta}{9(1-\delta)}$
  Player 1 will cooperate so long as $\delta \geq 0.5294$
- $\pi^2_{cooperating} \geq \pi^2_{cheating} \implies \frac{1250}{1-\delta} \geq 1406.25 + \frac{10000\delta}{9(1-\delta)}$
  Player 2 will cooperate so long as $\delta \geq 0.5294$

The trigger strategy will be a Nash equilibrium if $\delta \geq 0.5294$.